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Rational Functions

Unit 2 Lesson 5

Students will be able to:

Understand rational functions and properties associated with them

Key Vocabulary:

- Rational Function
- Domain, Range
- Vertical Asymptotes, x-intercepts
- Interpreting Graph of Rational Function

Rational Function

A rational function is a fraction of polynomials $p(x)$ and $q(x)$.

Mathematically:

$$f = \frac{p(x)}{q(x)}$$

Where $p(x)$ is the numerator and $q(x)$ is the denominator.

Example:

- $f = \frac{1}{x-1}$
- $f = \frac{y(y-2)}{(y-3)}$

Domain of Rational Function

Domain of a rational function is the set of all real numbers except the roots of the denominator polynomial $q(x)$.

Range of Rational Function

Range of a rational function is the set of all real numbers except those values of input(domain) that give the output as ∞ i.e. the output of the numbers excluded from domain.

Vertical Asymptotes

Vertical asymptotes are the vertical lines passing through the roots of denominator polynomial $q(x)$ and touching the graph of the rational function. The graph of rational function rises up or slides down the sides of the vertical asymptotes.

x-intercepts

These are the points where the graph of a rational function meets the x-axis and are the roots of the numerator polynomial $f(x)$ in the rational function.

Problem 1: Find the domain, vertical asymptotes and x-intercepts of the rational function $\frac{4(x-2)(x^2-1)}{3(x-3)(x+4)}$.

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Denominator has roots $x = 3$, $x = -4$.

Numerator has roots $x = 2, x = 1, x = -1$.

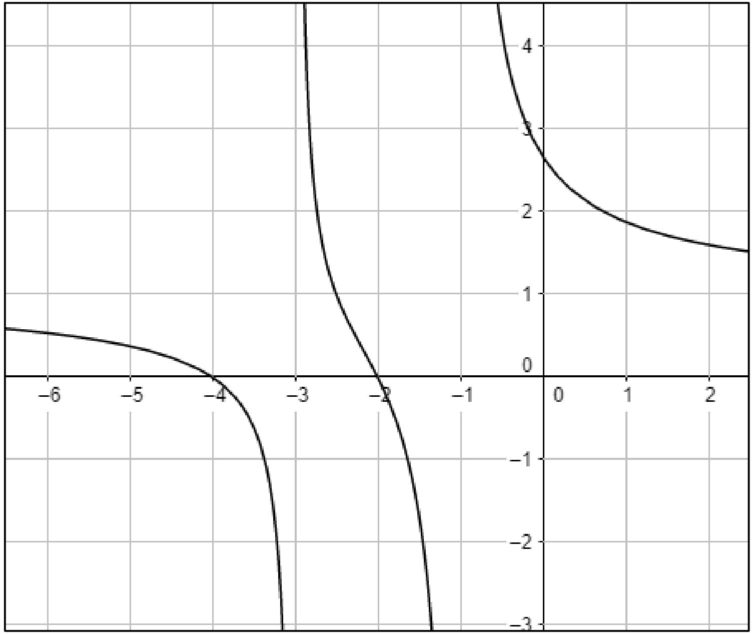
Domain: $R - \{3, -4\}$

Vertical Asymptotes: $x = 3, x = -4$

x-intercepts: $x = 2, x = 1, x = -1$

Interpreting Graph of Rational Function

Given the graph of a rational function, we can identify its vertical asymptotes and x-intercepts.



From the graph we can see that the function has two vertical asymptotes at $x = -3$ and $x = -1$ (since f is ∞).

Also, the graph touches x-axis at $x = -4$ and $x = -2$, which are the x-intercepts.

We can also say that numerator $p(x)$ has factors $(x + 4)(x + 2)$ and denominator $q(x)$ has factors $(x + 3)(x + 2)$.

Problem 2: Solve the equation $y + \frac{6}{y} = 5$.

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$$y\left(y + \frac{6}{y}\right) = y(5) \rightarrow y^2 + 6 = 5y$$

$$y^2 - 5y + 6 = 0$$

$$y^2 - 3y - 2y + 6 = 0$$

$$(y - 3)(y - 2) = 0$$

$$y = 3, y = 2$$