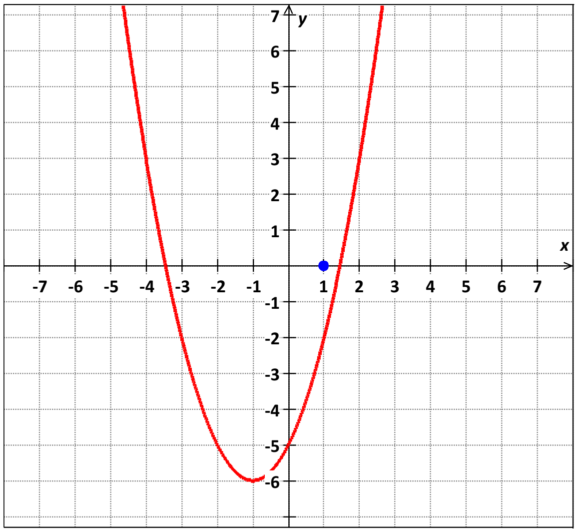
Name: ____ Period: ___________ Date: ________________

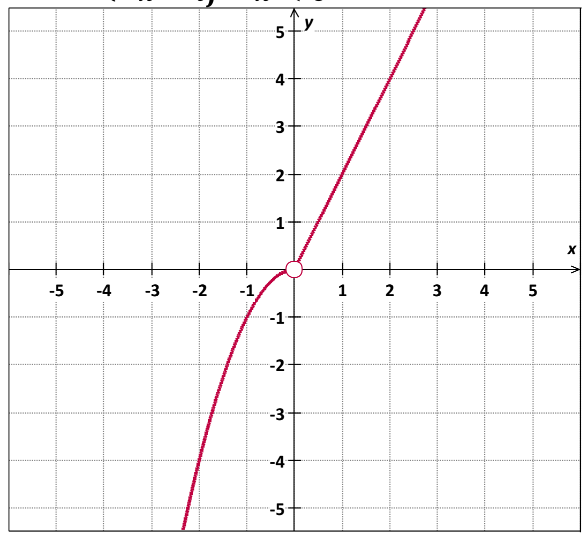
Continuity, End Behavior, and Limits Assignment

Determine whether each function is continuous at the given x-values. Justify using the continuity test. If discontinuous, identify the type of discontinuity as infinite, jump, or removable.

1. $f(x) = x^2 + 2x - 5 \quad at \quad x = 1$



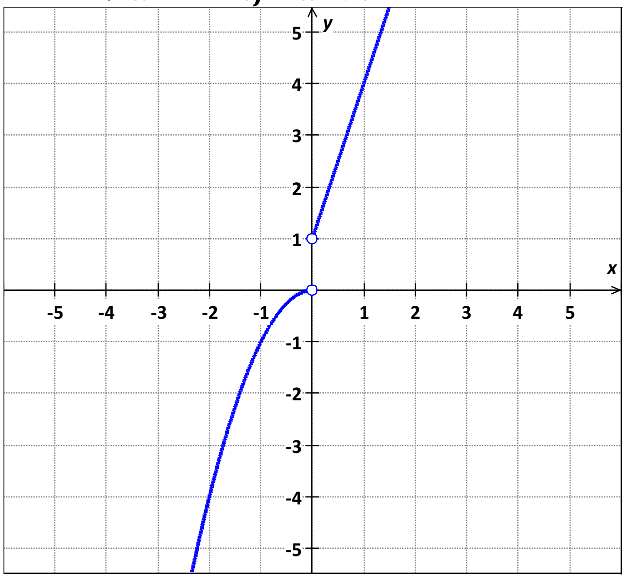
2. $f(x) = \begin{cases} 2x & if \quad x > 0 \\ -x^2 & if \quad x < 0 \end{cases} \quad x = 0$



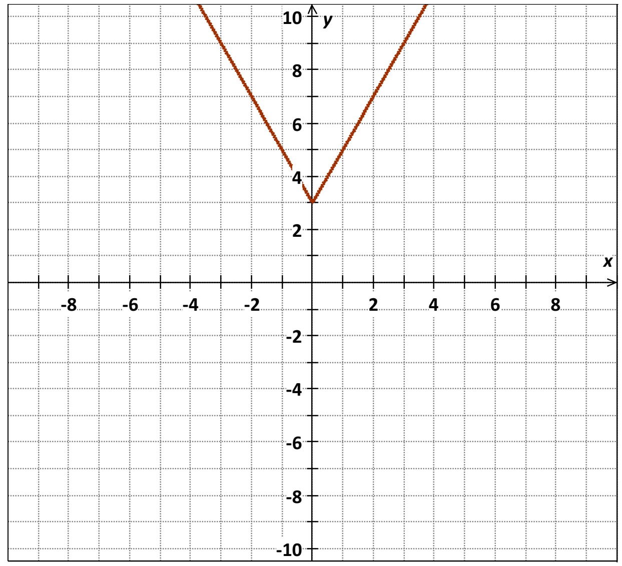
Name: ____ Period: __________ Date: _______________

Continuity, End Behavior, and Limits Assignment

3. $f(x) = \begin{cases} 3x + 1 & \text{if} \quad x \geq 0 \\ -x^2 & \text{if} \quad x < 0 \end{cases}$ $at \quad x = 0$



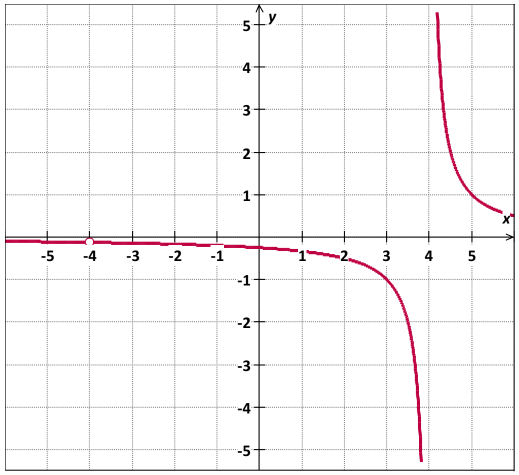
4. $f(x) = 2|x| + 3 \quad at \quad x = 2$



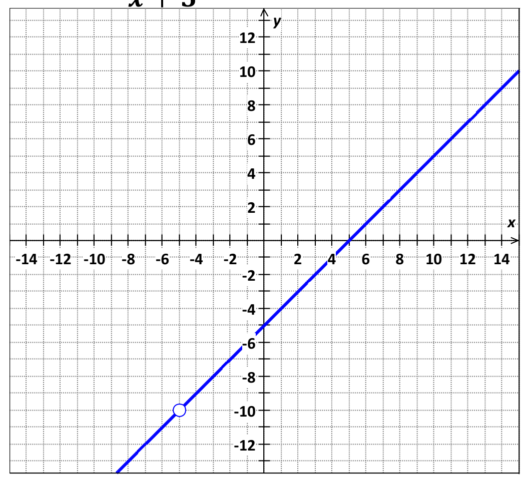
Name: ____ Period: __________ Date: _______________

Continuity, End Behavior, and Limits Assignment

5. $f(x) = \dfrac{x+4}{x^2-16}$ at $x = -4$ and $x = 4$



6. $f(x) = \dfrac{x^2-25}{x+5}$ at $x = -5$ and $x = 5$



Name: ____ Period: __________ Date: ______________

Continuity, End Behavior, and Limits Assignment

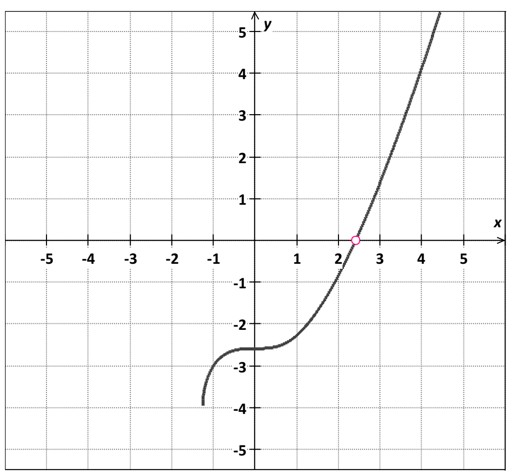
Find the value of k so that $f(x)$ is continuous.

7. $f(x) = \begin{cases} 8kx + 1 & if\ x > 3 \\ 2x + 5k & if\ x \leq 3 \end{cases}$

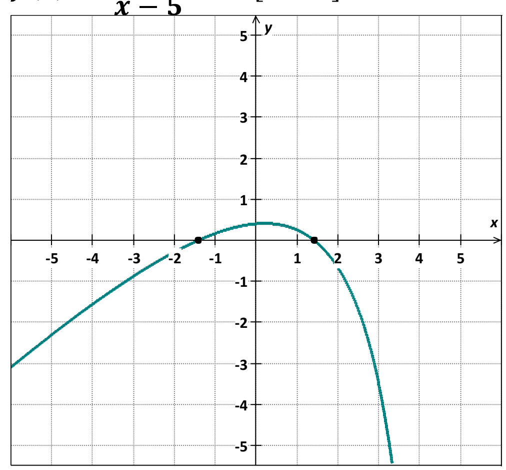
8. $f(x) = \begin{cases} 2x^2 + 1 & if & x > 0 \\ k & if & x = 0 \\ \frac{1}{2}x + 1 & if & x < 0 \end{cases}$

Determine between which consecutive integers the real zeros of function are located on the given interval.

9. $f(x) = \sqrt{x^3 + 2} - 4 \quad [0, 4]$



10. $f(x) = \dfrac{x^2 - 2}{x - 5} \quad [-2, 2]$

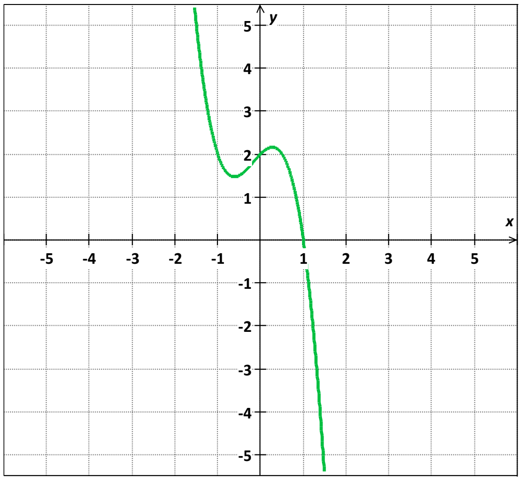


Name: ____ Period: __________ Date: _______________

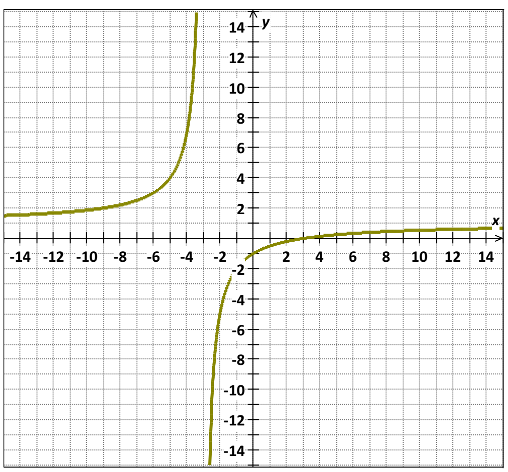
Continuity, End Behavior, and Limits Assignment

Use the graph of each function to describe its end behavior. Support the conjecture numerically.

11. $f(x) = -2x^3 - x^2 + x + 2$



12. $f(x) = \dfrac{x-3}{x+3}$



Evaluate the following limits.

13. $\lim\limits_{x \to 2} = \dfrac{x^2 - 4}{x - 2} = ?$

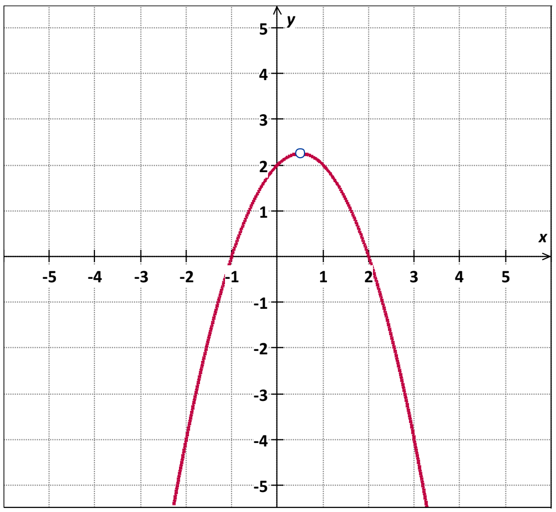
14. $\lim\limits_{x \to 1} -x^2 + 2x + 7 = ?$

Name: ____ Period: __________ Date: _______________

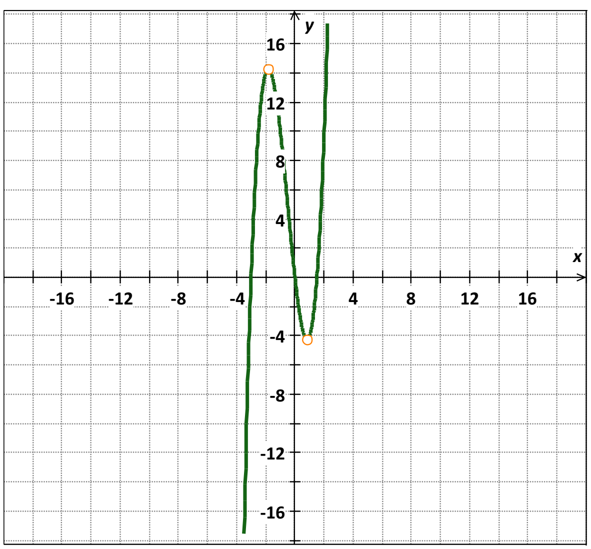
Continuity, End Behavior, and Limits Assignment

Determine the interval(s) on which the function is increasing and the interval(s) on which the function is decreasing.

15. $f(x) = -x^2 + x + 2$



16. $f(x) = 2x^3 + 3x^2 - 9x$



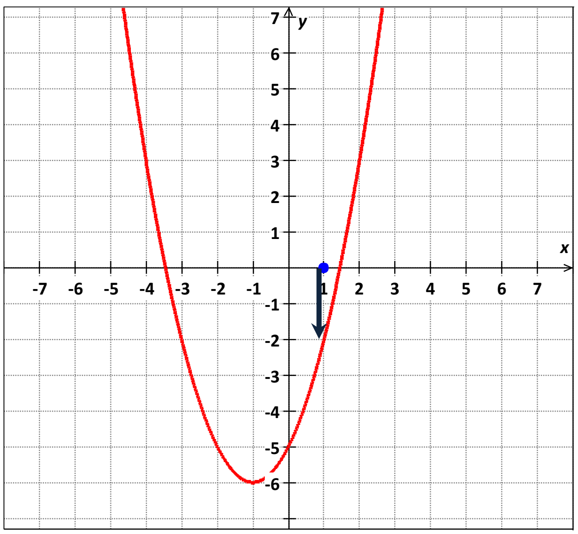
Name: ____ Period: ___________ Date: ________________

Continuity, End Behavior, and Limits Assignment

ANSWERS

Determine whether each function is continuous at the given x -values. Justify using the continuity test. If discontinuous, identify the type of discontinuity as infinite, jump, or removable.

1. $f(x) = x^2 + 2x - 5 \quad at \quad x = 1$



$f(x) = x^2 + 2x - 5 \quad at \quad x = 1$

$f(1) = 1^2 + 2 * 1 - 5$

$f(1) = -2 \qquad f(1)\ exists$

$x \to 1^- \quad y \to -2$

x	0.9	0.99	0.999
$f(x)$	-2.39	-2.0399	-2.00399

$x \to 1^+ \quad y \to -2$

x	1.1	1.01	1.001
$f(x)$	-1.59	-1.9599	-1.99599

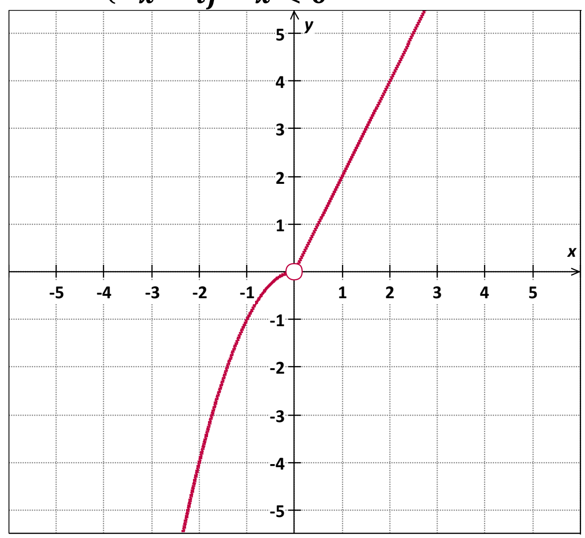
$f(1) = -2$

$and\ \ y \to -2\ \ from\ both\ side\ of\ x = 1$

$\lim_{x \to 1} x^2 + 2x - 5 = f(1)$

$f(x) = x^2 + 2x - 5\ \ is\ continuous\ at\ x = 1$

2. $f(x) = \begin{cases} 2x & if \quad x > 0 \\ -x^2 & if \quad x < 0 \end{cases} \quad x = 0$



$f(x) = \begin{cases} 2x & if \quad x > 0 \\ -x^2 & if \quad x < 0 \end{cases} \quad x = 0$

$f(x)\ is\ undefined\ in\ \ x = 0$

$f(x) = \begin{cases} 2x & if \quad x > 0 \\ -x^2 & if \quad x < 0 \end{cases}$

$is\ discontinuous\ at\ x = 0$

$x \to 0^- \quad -x^2 \to 0$

x	-0.1	-0.01	-0.001
$f(x)$	-0.01	-0.0001	-0.000001

$x \to 0^+ \qquad 2x \to 0$

x	0.1	0.01	0.001
$f(x)$	0.01	0.0001	0.000001

$f(x) = \begin{cases} 2x & if \quad x > 0 \\ -x^2 & if \quad x < 0 \end{cases}\ \ has\ point\ discontinuity$

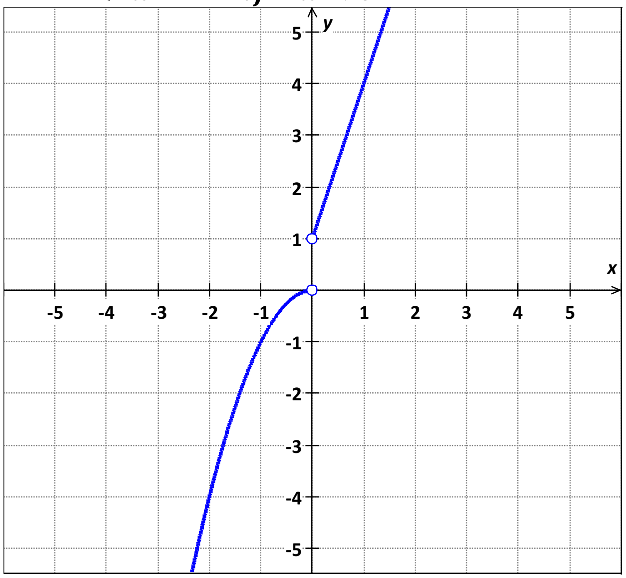
$since\ \ y \ \to\ 0\ \ on\ opposite\ sides\ of\ x = 0$

$x = 0\ \ is\ point\ o$

Name: ____ Period: __________ Date: _______________

Continuity, End Behavior, and Limits Assignment

3. $f(x) = \begin{cases} 3x+1 & if \quad x \geq 0 \\ -x^2 & if \quad x < 0 \end{cases}$ $at \quad x = 0$



$f(x) = \begin{cases} 3x+1 & if \quad x \geq 0 \\ -x^2 & if \quad x < 0 \end{cases}$ $at\ x = 0$

$f(0) = 3x + 1 = 3 * 0 + 1 = 1$
$f(0) \quad exists$

$x \to 0^- \quad -x^2 \to 0$

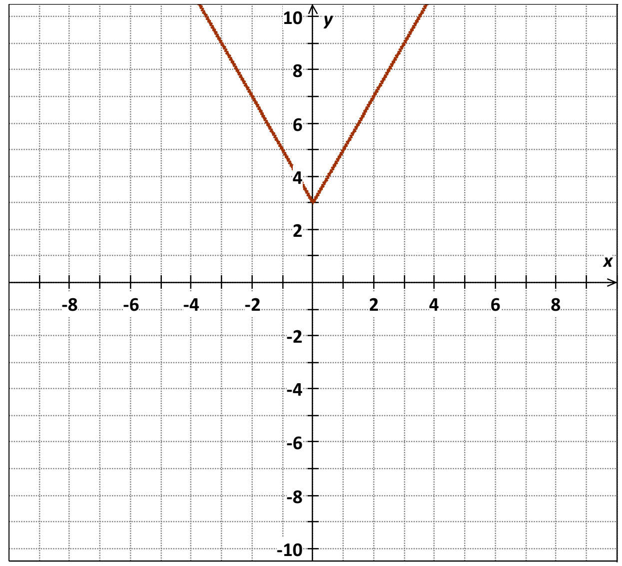
x	-0.1	-0.01	-0.001
$f(x)$	-0.01	-0.0001	-0.000001

$x \to 0^+ \qquad 3x + 1 \to 1$

x	0.1	0.01	0.001
$f(x)$	1.3	1.03	1.003

$f(x)$ *has jump discontinuity at* $x = 0$

4. $f(x) = 2|x| + 3 \quad at \quad x = 2$



$f(x) = 2|x| + 3 \quad at \quad x = 2$

$f(2) = 2|2| + 3$

$f(2) = 7 \qquad f(2)\ exists$

$x \to 2^- \quad y \to 7$

x	1.9	1.99	1.999
$f(x)$	6.8	6.98	6.998

$x \to 2^+ \quad y \to 7$

x	2.1	2.01	2.001
$f(x)$	7.2	7.02	7.002

$f(2) = 7$

and $y \to 7$ *from both side of* $x = 2$

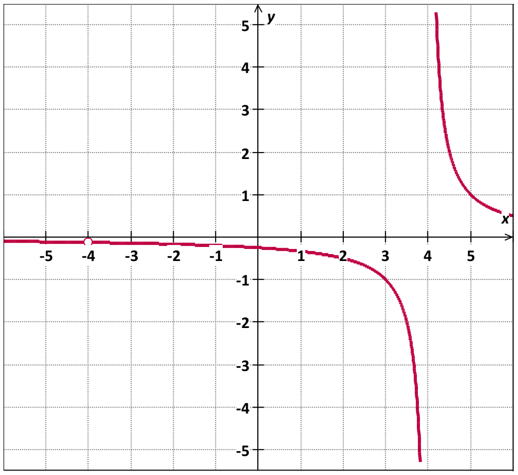
$\lim_{x \to 2} 2|x| + 3 \quad = f(2)$

$f(x) = 2|x| + 3 \quad$ *is continuous at* $x = 2$

Name: ____ Period: __________ Date: _______________

Continuity, End Behavior, and Limits Assignment

5. $f(x) = \dfrac{x+4}{x^2-16}$ at $x = -4$ and $x = 4$



$f(x) = \dfrac{x+4}{x^2-16}$ at $x = -4$ and $x = 4$

$f(-4) = \dfrac{(-4)+4}{(-4)^2-16} = \dfrac{0}{0}$

$f(4) = \dfrac{4+4}{(4)^2-16} = \dfrac{8}{0}$

$f(4)$ *and* $f(-4)$ *are undefined*

$x \to -4^- \quad y \to \approx \ -0.1266$

x	-4.1	-4.01	-4.001
$f(x)$	-0.1234	-0.1248	-0.1249

$x \to -4^+ \quad y \to \approx -0.1266$

x	-3.9	-3.99	-3.99
$f(x)$	-0.1266	-0.1266	-0.1251

$\lim_{x \to -4} f(x) \approx -0.1266$

$x \to 4^- \quad y \to \infty$

x	3.9	3.99	3.999
$f(x)$	-10	-100	-1000

$x \to 4^+ \quad y \to \infty$

x	4.1	4.01	4.001
$f(x)$	10	100	$-1{,}000$

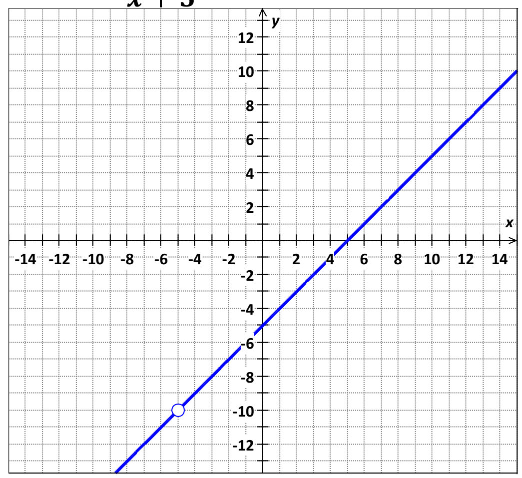
$\lim_{x \to 4} f(x)$ ***does not exists***

$\lim_{x \to -4} f(x) \approx -0.1266$ but $f(-4)$ is undefined

$f(x)$ has a removable discontinuity at $x = -4$

$f(x)$ has an infinite discontinuity at $x = 4$

6. $f(x) = \dfrac{x^2-25}{x+5}$ at $x = -5$ and $x = 5$



$f(x) = \dfrac{x^2-25}{x+5}$ at $x = -5$ and $x = 5$

$f(-5) = \dfrac{(-5)^2-25}{-5+5} = \dfrac{0}{0}$

$f(5) = \dfrac{(5)^2-25}{5+5} = \dfrac{0}{10} = 0$

$f(-5)$ *undefined*

$x \to -5^- \quad y \to -10$

x	-5.1	-5.01	-5.001
$f(x)$	-10.1	-10.01	-10.001

$x \to -5^+ \quad y \to -10$

x	-4.9	-4.99	-4.99
$f(x)$	-9.9	-9.99	-9.999

$\lim_{x \to -5} f(x) = -10$

$x \to 5^- \quad y \to 0$

x	4.9	4.99	4.999
$f(x)$	-0.1	-0.01	-0.001

$x \to 5^+ \quad y \to 0$

x	5.1	5.01	5.001
$f(x)$	0.1	0.01	0.001

$\lim_{x \to 5} f(x) = f(5) = 0$

$\lim_{x \to -5} f(x)$ ***exists*** but $f(-5)$ is undefined

$f(x)$ has a removable discontinuity at $x = -5$

$\lim_{x \to 5} f(x) = f(5) = 0$

$f(x)$ ***is continuous at*** $x = 5$

Name: ____ Period: __________ Date: _______________

Continuity, End Behavior, and Limits Assignment

Find the value of k so that $f(x)$ is continuous.

7. $f(x) = \begin{cases} 8kx + 1 & if\ x > 3 \\ 2x + 5k & if\ x \le 3 \end{cases}$

$8kx + 1 = 2x + 5k \qquad x = 3$

$8k * 3 + 1 = 2 * 3 + 5k$

$24k + 1 = 6 + 5k$

$19k = 5$

$k = \frac{5}{19}$

8. $f(x) = \begin{cases} 2x^2 + 1 & if \quad x > 0 \\ k & if \quad x = 0 \\ \frac{1}{2}x + 1 & if \quad x < 0 \end{cases}$

$2x^2 + 1 = \frac{1}{2}x + 1 \qquad x = 0$

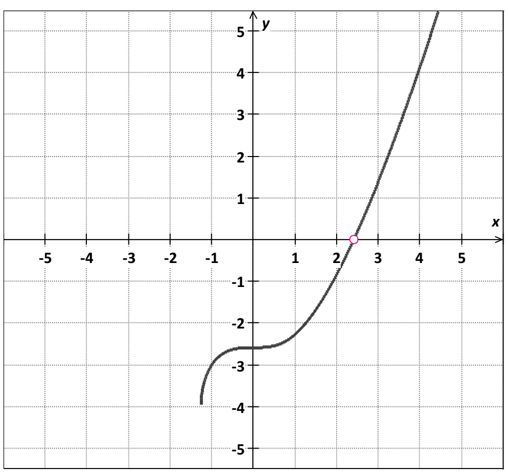
$2 * 0^2 + 1 = \frac{1}{2} * +1$

$1 = 1$

$k = 1$

Determine between which consecutive integers the real zeros of function are located on the given interval.

9. $f(x) = \sqrt{x^3 + 2} - 4 \qquad [0, 4]$

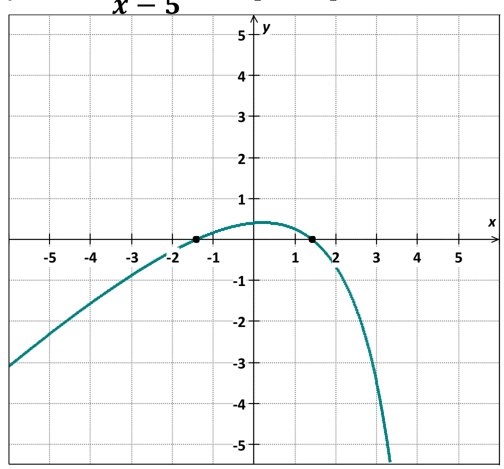


x	0	1	2	3	4
y	-2.58	-2.26	-0.83	1.38	4.12

$f(2)$ is negative positive and $f(3)$ is positive,
$f(x)$ ***change sign in*** $2 \le x \le 3$

$f(x)$ has zero in interval: $2 \le x \le 3$

10. $f(x) = \frac{x^2 - 2}{x - 5} \qquad [-2, 2]$



x	-2	-1	0	1	2
y	$-\frac{2}{7}$	$\frac{1}{6}$	$\frac{2}{5}$	$\frac{1}{4}$	$-\frac{2}{3}$

$f(-2)$ is negative and $f(-1)$ is positive,
$f(x)$ ***change sign in*** $-2 \le x \le -1$
$f(1)$ is positive and $f(2)$ is negative
$f(x)$ ***change sign in*** $1 \le x \le 2$

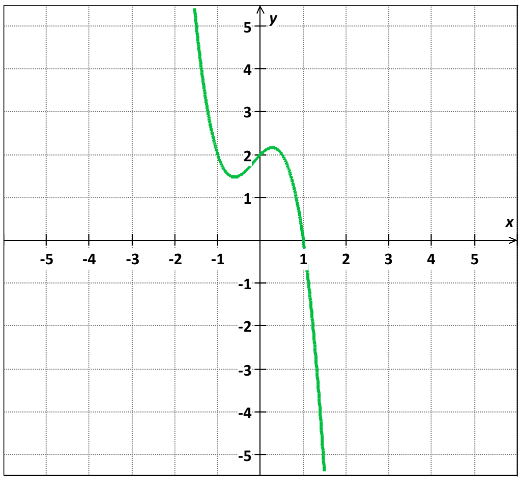
$f(x)$ has zeros in intervals:
$-2 \le x \le -1 \quad and \quad 1 \le x \le 2$

Name: ____ Period: __________ Date: _______________

Continuity, End Behavior, and Limits Assignment

Use the graph of each function to describe its end behavior. Support the conjecture numerically.

11. $f(x) = -2x^3 - x^2 + x + 2$



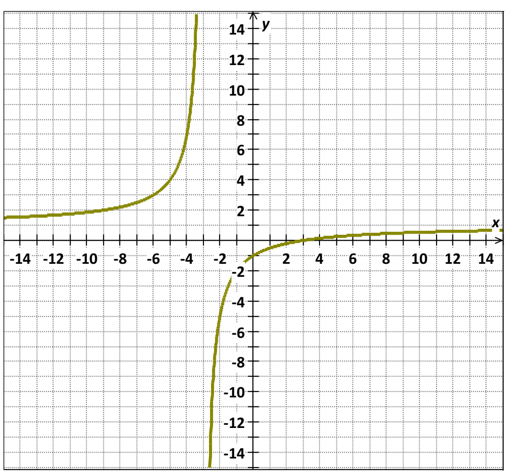
From the graph, it appears that:

$f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$

The table supports this conjecture.

x	-10^4	-10^3	0	10^3	10^4
y	$2 * 10^{12}$	$2 * 10^9$	2	$-2 * 10^9$	$-2 * 10^{12}$

12. $f(x) = \dfrac{x-3}{x+3}$



From the graph, it appears that:

$f(x) \to 1$ as $x \to -\infty$ and $f(x) \to 1$ as $x \to \infty$

The table supports this conjecture.

x	-10^4	-10^3	0	10^3	10^4
y	1	1	-1	1	1

Evaluate the following limits.

13. $\lim_{x \to 2} = \dfrac{x^2 - 4}{x - 2} = ?$

14. $\lim_{x \to 1} -x^2 + 2x + 7 = ?$

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Name: ____ Period: __________ Date: _______________

Continuity, End Behavior, and Limits Assignment

$\lim_{x\to 2} = \frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2}$

$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2}(x+2) = 2+2 = 4$

$\lim_{x\to 2} x+2 = 4$

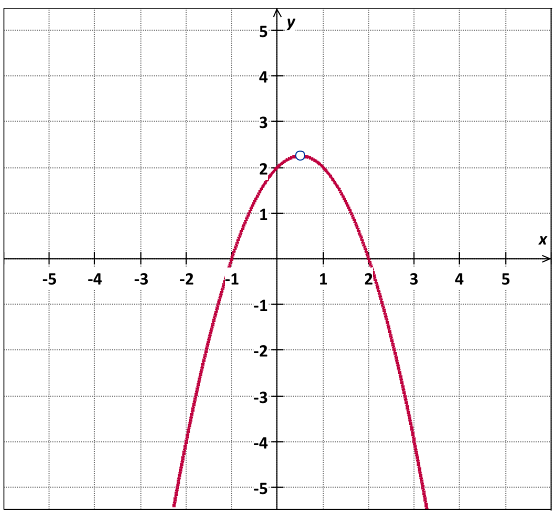
$\lim_{x\to 1} -x^2+2x+7 = -1^2+2*1+7$

$\lim_{x\to 1} -x^2+2x+7 = -1+2+7$

$\lim_{x\to 1} -x^2+2x+7 = 8$

Determine the interval(s) on which the function is increasing and the interval(s) on which the function is decreasing.

15. $f(x) = -x^2+x+2$



From the graph, it appears that:

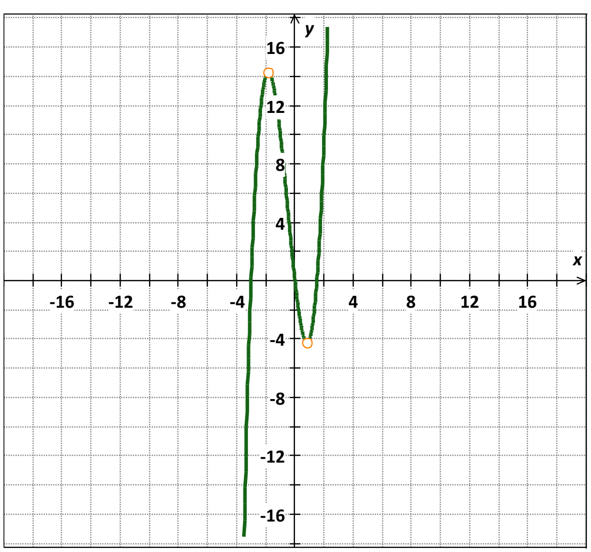
A function $-x^2+x+2$ is increasing for $x < 0.5$

A function $-x^2+x+2$ is decreasing for $x > 0.5$

The table supports this conjecture.

x	-1	0	0.5	1	2
y	0	2	2.25	2	0

16. $f(x) = 2x^3+3x^2-9x$



From the graph, it appears that:

A function $2x^3+3x^2-9x$ is increasing for $x < -1.82$

A function $2x^3+3x^2-9x$ is decreasing for

$-1.82 < x < 0.82$

A function $2x^3+3x^2-9x$ s increasing for $x > 0.82$

The table supports this conjecture.

x	-3	-1.82	0	0.82	1
y	0	14.26	0	-4.26	-4

Name: ____ Period: ___________ Date: ________________

Continuity, End Behavior, and Limits Assignment