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Operations with Complex Numbers

Unit 0 Lesson 2

OPERATIONS WITH COMPLEX NUMBERS

Students will be able to:

Understand the definition of complex numbers and simplify complex numbers involving mathematical operations

Key Vocabulary:

- Complex Numbers
- Add and Subtract Complex Numbers
- Multiply Complex Numbers
- Complex Conjugates
- Divide Complex Numbers

Complex Number

A complex is any number that can be written in the form:

$$a + bi$$

Where a and b are Real numbers and $i = \sqrt{-1}$.

Here,

$a = Real\ Part$

$bi = Imaginary\ Part$

$$i \times i = \sqrt{-1} \times \sqrt{-1}$$
$$i^2 = \sqrt{-1 \times -1} = \sqrt{-1^2}$$
$$i^2 = -1$$

OPERATIONS WITH COMPLEX NUMBERS

Problem 1: Write the following as a complex number.

(a) -4

(b) $\sqrt{-36}$

(c) $-8+\sqrt{-25}$

Problem 1: Write the following as a complex number.

(a) -4

$$-4 = -4 + 0i$$

(b) $\sqrt{-36}$

$$\sqrt{-36} = \sqrt{-1} \times \sqrt{36}$$

$$\sqrt{-36} = 6i$$

(c) $-8 + \sqrt{-25}$

$$-8 + \sqrt{-25} = -8 + \sqrt{-1} \times 5$$

$$-8 + \sqrt{-25} = -8 + 5i$$

Adding and Subtracting Complex Numbers

Adding and subtracting complex numbers is similar to adding and subtracting polynomials. We add/subtract the real parts to real parts and imaginary parts to imaginary parts.

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

Multiplying Complex Numbers

Multiplying complex numbers is similar to multiplying polynomials. We can use the expression below in doing a quick multiplication.

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

Problem 2: Simplify the following:

a) $(5 + 2i) + (-2 + 3i)$

b) $(-6 - 2i) - (2 + 4i)$

c) $(2 + 3i)(6 + 4i)$

Problem 2: Simplify the following:

a) $(5 + 2i) + (-2 + 3i)$

$$5 + (-2) + (2 + 3)i = 3 + 5i$$

b) $(-6 - 2i) - (2 + 4i)$

$$-6 - 2 + (-2 - 4)i = -8 - 6i$$

c) $(2 + 3i)(6 + 4i)$

$$\big(2(6) - 3(4)\big) + \big(2(4) + 3(6)\big)i = (12 - 12) + (8 + 18)i$$

$$(12 - 12) + (8 + 18)i = 0 + 26i = 26i$$

Complex Conjugates

The numbers $a + bi$ and $a - bi$ are called the complex conjugates of each other.

Dividing Complex Numbers

Dividing complex numbers is similar to the rationalization process i.e. we multiply and divide the fraction with the complex conjugate of the denominator, so that the resulting fraction does not have i in the denominator.

Problem 3: Write the quotient $\frac{3-i}{3+i}$ in the form $a + bi$.

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We multiply and divide the fraction with the complex conjugate of $3 + i$ which is $3 - i$.

$$\frac{3-i}{3+i} \times \frac{3-i}{3-i} = \frac{(3-i)^2}{3^2-i^2}$$

$$\frac{(3-i)^2}{3^2-i^2} = \frac{9+i^2-2(3)i}{9-(-1)}$$

$$\frac{9+i^2-2(3)i}{9-(-1)} = \frac{9-1-6i}{10} = \frac{8-6i}{10} = \frac{4}{5} - \frac{3i}{5}$$