

Unit 7 Lesson 4

## Students will be able to:

- Define mean, variability and mean absolute deviation.
- Describe the center of distribution by interpreting the mean as "fair share" and as a "balancing point".
- Calculate the mean of a given set of data.
- Compare data sets with the same mean but different variability.
- Calculate the mean absolute deviation of a given set of data.
- Describe the center and variability of a distribution using the mean absolute deviation.



# **Key Vocabulary:**

Data Distribution

Mean

Fair Share

**Balancing Point** 

Variability

**Absolute Deviation** 

Mean Absolute Deviation

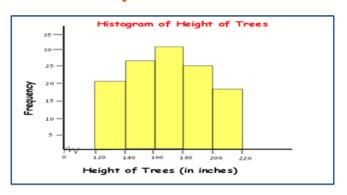


## The Center of Data Distribution

In the past lessons, you were asked to describe the center of the data distribution given dot plots and histograms. We only determined the middle most value of a data distribution to describe the center of the distribution. This lesson will focus on the in-depth explanation of the concept of "center" especially in data distributions.

## The Mean as a Measure of Central Tendency

The "mean" or "average" (in simple terms) is the most appropriate way to describe and summarize data distributions that are approximately symmetric. Here we are trying to find that single number that represents the entire set of data.

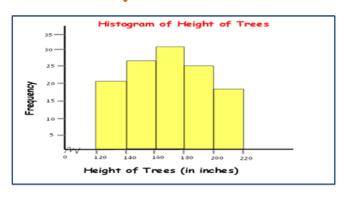




## The Mean as a Measure of Central Tendency

There are other ways to find the center of the distribution though; you'll learn more of these in the next lessons. In the meantime, let's concentrate in finding the "mean".

Before jumping into the "calculation" part, let's determine the mean by using "Fair Share".





## Interpreting the Mean as Fair Share

What does "fair share" mean?

Those that have the most, give something to those with the least; until everyone has exactly the same amount.

Look at the dot plot on then next slide and let's see how "fair share" is done.

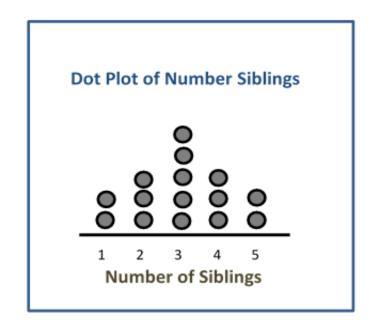


## Interpreting the Mean as Fair Share

Sheena wants to know the typical number of siblings her five friends have.

Below are the data she collected and on the right is the dot plot that displays these data.

4, 4, 3, 1, 5, 3, 3, 1, 3, 2, 5, 2, 4, 2, 3



## Interpreting the Mean as Fair Share

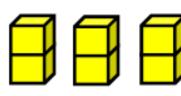
Let's stack some cubes to show how "fair share" is done.

Remember the data: 4, 4, 3, 1, 5, 3, 3, 1, 3, 2, 5, 2, 4, 2, 3

Two students have 1 sibling:

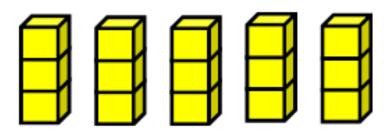


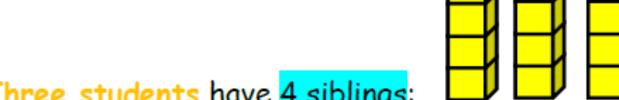
Three students have 2 siblings:



Remember the data: 4.4.3.1.5.3.3.1.3.2.5.2.4.2,3

Five students have 3 siblings:



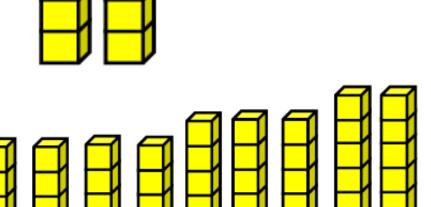


Three students have 4 siblings:

## Interpreting the Mean as Fair Share

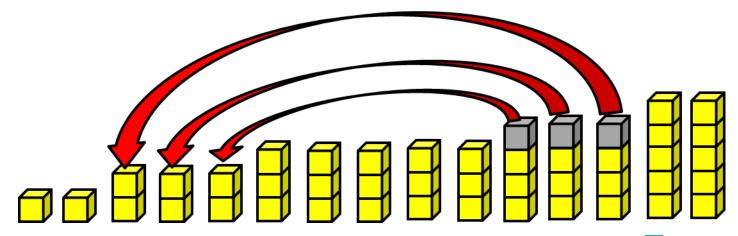
Remember the data: 4, 4, 3, 1, 5, 3, 3, 1, 3, 2, 5, 2, 4, 2, 3

Two students have 5 siblings:

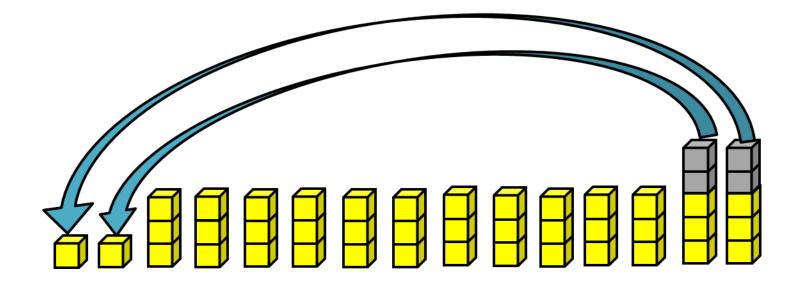


Now, it's time to share!

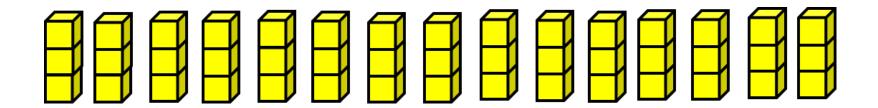
Those that have the most, give something to those with the least; until everyone has exactly the same amount.



## Interpreting the Mean as Fair Share



Now, everyone has exactly the same number of cubes.



This is how we interpret the mean as "fair share". Now each one has 3 cubes. This means that the center of the distribution is 3.

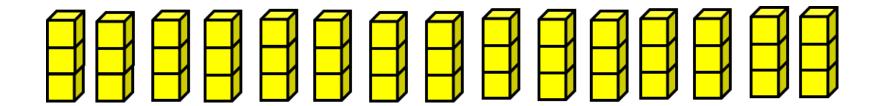
This means that the mean or average number of siblings for each student is 3. Also, 3 is the single number that represents the given set of data.

Sheena wants to know the typical number siblings her five friends have.

Below are the data she collected

4, 4, 3, 1, 5, 3, 3, 1, 3, 2, 5, 2, 4, 2, 3

The typical number of siblings Sheena's friends have is 3.



## Interpreting the Mean as Fair Share

Let's have another example of interpreting the mean as "fair Share".

Tom wants to know his mean score in his five Statistics tests.

Below are his scores. 88, 86, 94, 92, 90

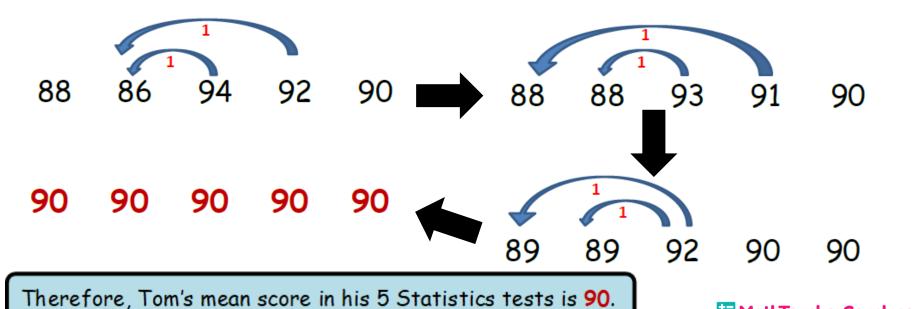
Interpret the mean as "fair share".



## Interpreting the Mean as Fair Share

Remember:

Those that have the most, give something to those with the least; until everyone has exactly the same amount.



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Sample Problem 1: Read the problem and interpret the mean as fair share.

 Liza wants to know the typical number of text messages five of her friends send in a day.

Below are the data she collected 4, 3, 6, 8, 4

Using cubes, interpret the mean as "fair share".





Sample Problem 1: Read the problem and interpret the mean as fair share.

Solution:

## Sample Problem 1: Solution.

 Liza wants to know the typical number of text messages five of her friends send in a day.

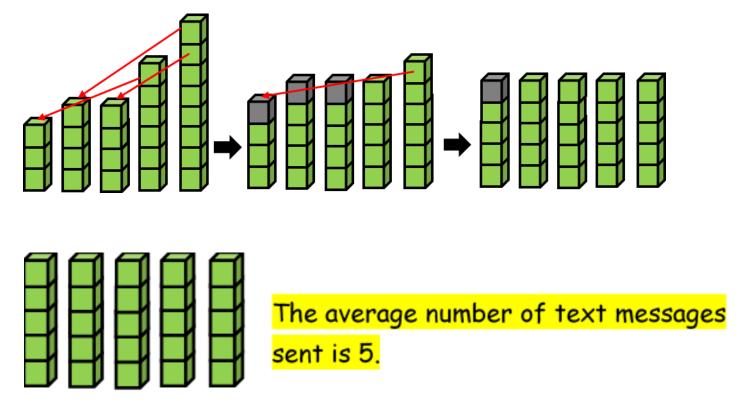
Below are the data she collected 4, 3, 6, 8, 4

Using cubes, interpret the mean as "fair share".





## Sample Problem 1: Solution



Sample Problem 1: Read the problem and interpret the mean as fair share.

2. Stanley wants to know the typical amount of milk (in Liters) his 5 cows produce in a day.

Below are the data he collected:

24, 27, 30, 26, 23

Interpret the mean as "fair share".



Sample Problem 1: Read the problem and interpret the mean as fair share.

Solution:

## Sample Problem 1: Solution

2. Stanley wants to know the typical amount of milk (in Liters) his 5 cows produce in a day.

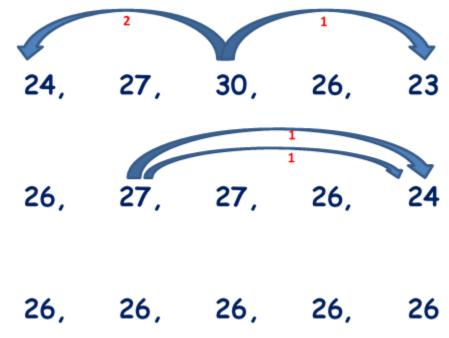
Below are the data he collected:

24, 27, 30, 26, 23

Interpret the mean as "fair share".



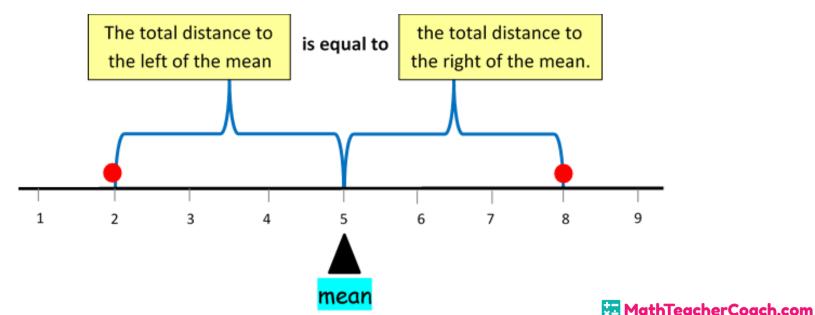
## Sample Problem 1: Solution



The average milk that the cows produce (in Liters) is 26.

## Mean as a Balancing Point

To interpret the mean as a balancing point, we need to understand that the following distances from the mean are equal:



## Mean as a Balancing Point

Look at the example below and find the mean.

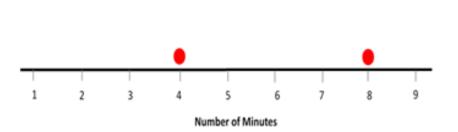
Below is the dot plot that shows the number of minutes it takes for two students to walk home from school. Dot Plot of Number of Minutes Number of Minutes



## Mean as a Balancing Point

1. Where should the balance point be?

Remember that the balance point represents the mean of the data. Also, the total distances to the left of the balancing point must be equal to the total distances to its right.



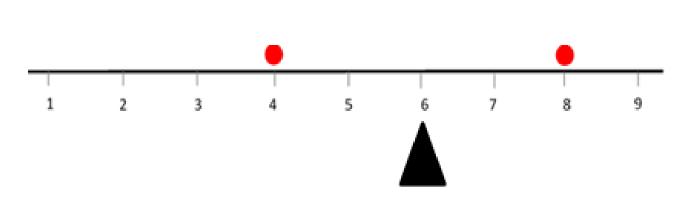
Dot Plot of Number of Minutes

## Mean as a Balancing Point

In the dot plot on the right, the balancing point must be at 6.

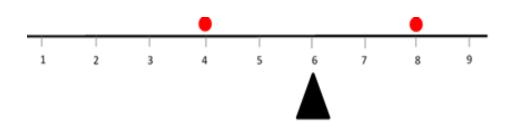
Dot Plot of Number of Minutes

**Number of Minutes** 





## Mean as a Balancing Point



The distance to the left of the balancing point, between 3 and 6 is 3.

The distance to the right of the balancing point, between 3 and 8 is 3.

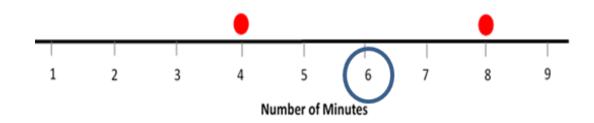


## Mean as a Balancing Point

2. What is the mean?

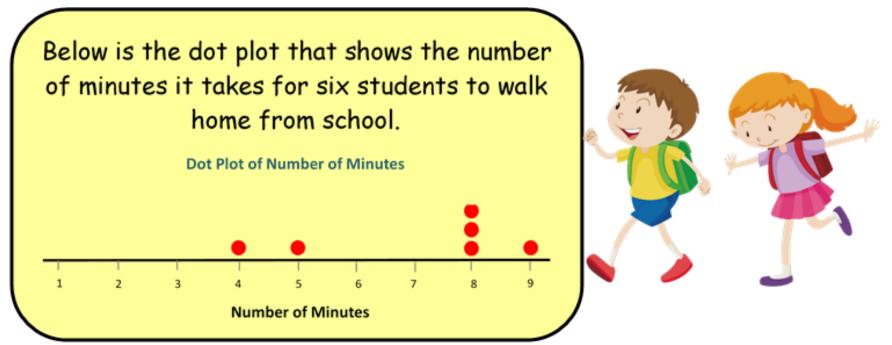
Dot Plot of Number of Minutes

The mean is 6.



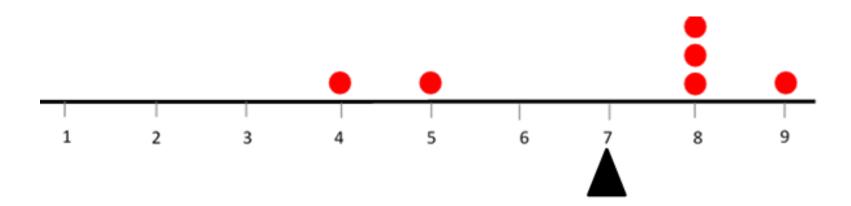
## Mean as a Balancing Point

If we add more data to the problem, will the mean change?

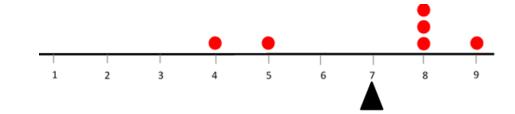


## Mean as a Balancing Point

To have equal total distances to the left and to the right of the mean, the balance point must be at 7. Therefore, the mean must be 7.



# Mean as a Balancing Point

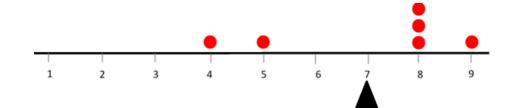


One of the distances to the left of the mean (between 5 and 7) is 2.

One of the distances to the left of the mean (between 4 and 7) is 3.

Total distance to the left of the mean: 2 + 3 = 5

# Mean as a Balancing Point

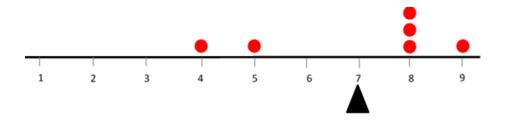


One of the distances to the right of the mean (between 7 and 8) is 1. Since there are three data point at 8, we'll count the distance between 7 and 8 three times:  $\mathbf{1} + \mathbf{1} + \mathbf{1} = \mathbf{3}$ 

One of the distances to the right of the mean (between 7 and 9) is 2.

Total distance to the right of the mean: 3 + 2 = 5

# Mean as a Balancing Point



Now, we can really say that the value that represents the typical number of minutes six students walk home form school is 7. This means that the mean of the given data is 7.

Sample Problem 2: Display the data using a dot plot, find the balancing point to determine the mean.

Sheena wants to know the typical number of pets her twelve friends have.

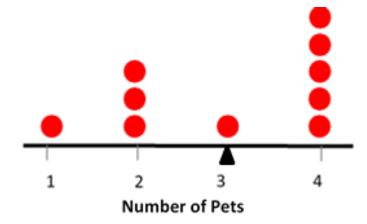
Below are the data she collected:

4, 1, 2, 4, 2, 4, 2, 4, 3, 4,



## Sample Problem 2: Solution

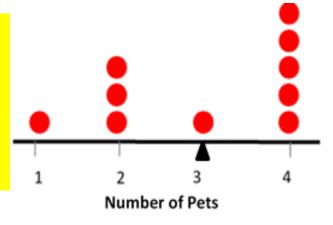
To have equal total distances to the left and to the right of the mean, the balance point must be at 3. Therefore, the mean must be 3.



# Sample Problem 2: Solution

three times: 1 + 1 + 1 = 3

One of the distances to the left of the mean (between 3 and 2) is 1. Since there are three data point at 2, we'll count the distance between 3 and 2



One of the distances to the left of the mean (between 3 and 1) is 2.

Total distance to the left of the mean: 3 + 2 = 5

# Sample Problem 2: Solution

The distance to the right of the mean (between 3 and 4) is 1. Since there are five data point at 4, we'll count the distance between 3 and 4 five times: 1 + 1 + 1 + 1 + 1 = 5

1 2 3 4
Number of Pets

Therefore, the mean is 5.

## Calculating the Mean

Aside from using "fair share" and "balancing point" to determine the mean of a given set of data, this too can be done mathematically by calculating it using a formula.

#### Formula for the mean:

$$mean = \frac{sum\ of\ all\ data}{number\ of\ observations}$$

# Calculating the Mean

Let's use the previous examples to check if the answers will remain the same. To get the mean of the data on the right, the concept of "fair share" was used.

Sheena wants to know the typical number of siblings her five friends have.

Below are the data she collected:

4, 4, 3, 1, 5, 3, 3, 1, 3, 2, 5, 2, 4, 2, 3

# Calculating the Mean

Sheena wants to know the typical number of siblings her five friends have.

Below are the data she collected:

4, 4, 3, 1, 5, 3, 3, 1, 3, 2, 5, 2, 4, 2, 3 We used the "fair share" method to determine the mean.

Here, the mean is 3.

Will the mean be the same if we calculate it using the formula?

Let's find it out!

## Calculating the Mean

Sheena wants to know the typical number of siblings her five friends have.

Below are the data she collected:

$$mean = \frac{sum \ of \ all \ data}{number \ of \ observations}$$

Sum of all data:

Add up all the data collected.

Number of observations:

Count the number of data you have.



# Calculating the Mean

$$mean = \frac{sum \ of \ all \ data}{number \ of \ observations}$$
 
$$mean = \frac{4+4+3+1+5+3+3+1+3+2+5+2+4+2+3}{15}$$
 
$$mean = \frac{45}{15}$$
 
$$mean = 3$$

So it is TRUE! The fair share method and the

formula gave us the same result!

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# Calculating the Mean

Let's try this one too!

Using the "fair share" method the mean is 90.

Will the mean be the same if we calculate it using the formula?

Let's find it out!

Tom wants to know his mean score in his five Statistics tests.

Below are his scores.

88, 86, 94, 92, 90

# Calculating the Mean

$$mean = \frac{sum \ of \ all \ data}{number \ of \ observations}$$

$$mean = \frac{88 + 86 + 94 + 92 + 90}{5}$$

$$mean = \frac{450}{5}$$

$$mean = 90$$

Tom wants to know his mean score in his five Statistics tests.

Below are his scores. 88, 86, 94, 92, 90

The methods may be different, but the results are still the same. The mean is 90.

Sample Problem 3: Find the mean for each set of data using the formula.

Basketball Points
41, 32, 45, 29, 30, 27
Mean = ?

Exam Scores			
92, 96, 94, 88, 88, 92, 87			
Mean = ?			

Sample Problem 3: Find the mean for each set of data using the formula.

Hours of Sleep	
10, 9, 13, 10, 12, 10, 8, 8, 10	
Mean = ?	

Number of Emails
15, 19, 19, 17, 18, 17, 16, 17, 15
Mean = ?

Sample Problem 3: Find the mean for each set of data using the formula.

Body Length (in cm)	Height of Students (in inches)
142.5, 137.25, 150.75, 139.5	57, 59, 56, 59, 62, 60, 58, 59, 57
Mean = ?	Mean = ?

## Sample Problem 3: Solution

# Basketball Points 41, 32, 45, 29, 30, 27 $mean = \frac{41 + 32 + 45 + 29 + 30 + 27}{6}$ $mean = \frac{204}{6}$ mean = 34

# Exam Scores 92, 96, 94, 88, 88, 92, 87 $mean = \frac{92 + 96 + 94 + 88 + 88 + 92 + 87}{7}$ $mean = \frac{637}{7}$ mean = 91

## Sample Problem 3: Solution

### Hours of Sleep

10, 9, 13, 10, 12, 10, 8, 8, 10

$$mean = \frac{10+9+13+10+12+10+8+8+10}{9}$$

$$mean = \frac{90}{9}$$

$$mean = 10$$

#### Number of Emails

15, 19, 19, 17, 18, 17, 16, 17, 15

$$mean = \frac{15 + 19 + 19 + 17 + 18 + 17 + 16 + 17 + 15}{6}$$

$$mean = \frac{153}{9}$$

$$mean = 17$$

## Sample Problem 3: Solution

#### Body Length (in cm)

142.5, 137.25, 150.75, 139.5

$$mean = \frac{142.5 + 137.25 + 150.75 + 139.5}{4}$$

$$mean = \frac{570}{4}$$

$$mean = 142.5$$

### Height of Students (in inches)

57, 59, 56, 59, 62, 60, 58, 59, 57

$$mean = \frac{57 + 59 + 56 + 59 + 62 + 60 + 58 + 59 + 57}{9}$$

$$mean = \frac{527}{9}$$

$$mean = 58.56$$

# The Variability in the Distribution

Variability in a distribution refers to how "spread out" or "scattered" the data around the mean. Sometimes, distributions may have the same mean but can have different variability. This measures how much the data differ from each other.

## The Variability in the Distribution

## There are two things you need to look out for:

#### 1. Are the data spread out around the mean?

In this case, there is a greater variability (wide spread) in the distribution. Thus, the mean is not a good representation of a typical value in a data set.

#### 2. Are the data clustered around the mean?

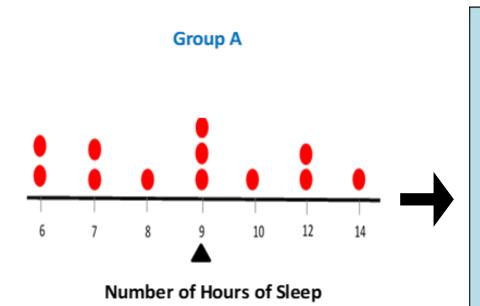
In this case, there is a lesser variability (closer to the mean) in the distribution. Thus, the mean indicates an accurate representation of a typical value in a data set.

## The Variability in the Distribution

The dot plots on the next slides show the number of hours students sleep during weekends. The data were taken for two different groups of students.

Both data set has the same mean, 9.

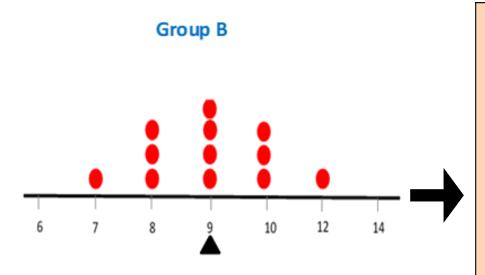
## The Variability in the Distribution



The data in Group A ranges from 6 hours to 14 hours. This shows a greater variability because they are spread out around the mean.

Thus, its mean which is 9 is not a good indicator of a typical number of hours students sleep on weekends.

## The Variability in the Distribution

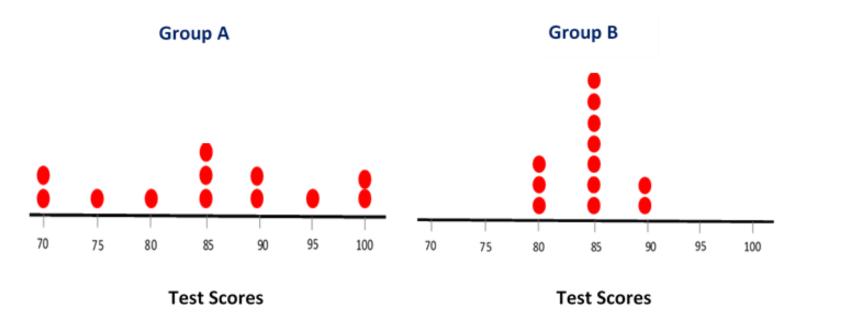


**Number of Hours of Sleep** 

The data in Group B ranges from 7 hours to 12 hours. This shows a lesser variability because they are clustered around the mean.

Thus, its mean which is 9 is an accurate indicator of a typical number of hours students sleep on weekends.

Sample Problem 4: Below are the dot plots of the scores in a Math test from two different groups. Analyze the dot plots and answer the questions that follow.



# Sample Problem 4:

## Questions:

1. What is the mean score for each group? Compute for the mean score. (Round of to a whole number if needed)

2. Which distribution has the mean that is a more accurate indicator of the typical test score?

# Sample Problem 4: Solution

## Questions:

1. What is the mean score for each group? Compute for the mean score. (Round off to a whole number if needed)

Group A	Group B
$mean = \frac{1025}{12}$	$mean = \frac{1015}{12}$
$mean \approx 85.42$	$mean \approx 84.58$
mean ≈ 85	mean ≈ 85

Sample Problem 4: Solution

## Questions:

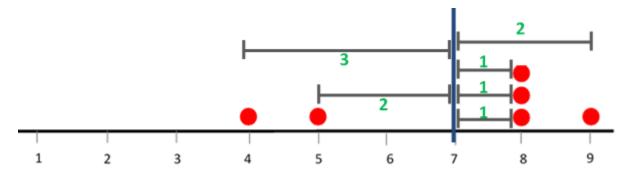
2. Which distribution has the mean that is a more accurate indicator of the typical test score?

Group B has lesser variability; therefore its mean is a more accurate indicator of the typical test score.

Before we discuss the mean absolute deviation, lets first understand what "absolute deviation" means.

Absolute deviation is the distance of a data value form the mean. To make it even simpler, it determines how far a data value is form the mean. Below is the dot plot that shows the number of minutes it takes for six students to walk home from school. Here, the mean is 7.

#### The Mean Absolute Deviation



Number of	Deviation from the Mean	Absolute Deviation
Minutes	(Distance and Direction)	(Distance form the Mean)
4	3 to the left	3
5	2 to the left	2
8	1 to the right	1
8	1 to the right	1
8	1 to the right	1
9	2 to the right	2

The total distances to the left of the mean is equal to the total distances to the to its right.

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The Mean Absolute Deviation (MAD) is the average of the absolute deviations, all the distances of the given data form the mean. Here's what the Mean Absolute Deviation tell us about the variability of a distribution.

 The value of the MAD tells us about average distance of the data values from the mean.

#### The Mean Absolute Deviation

 A smaller value of MAD tells us that the data distribution has very little variability. Also, the mean is an accurate indicator of a typical value in a distribution.

3. A larger value of MAD tells us that the data values are spread out and are far away from the mean. Also, the mean is not a good indicator of a typical value in a distribution.

To solve for the MAD for this set of data, here's what we need to do.

Number of	Deviation from the Mean	Absolute Deviation
Minutes	(Distance and Direction)	(Distance form the Mean)
4	3 to the left	3
5	2 to the left	2
8	1 to the right	1
8	1 to the right	1
8	1 to the right	1
9	2 to the right	2

This will be easier because we already know the mean. Find the sum of the absolute deviation:

Number of	Deviation from the Mean	Absolute Deviation
Minutes	(Distance and Direction)	(Distance form the Mean)
4	3 to the left	3
5	2 to the left	2
8	1 to the right	1
8	1 to the right	1
8	1 to the right	1
9	2 to the right	2
		Total = 10

To get the MAD, divide the sum of the absolute deviations and by the number of observations.

$$MAD = \frac{sum\ of\ the\ absolute\ deviations}{number\ of\ observations}$$
  $MAD = \frac{10}{6}$   $MAD \approx 1.67$ 

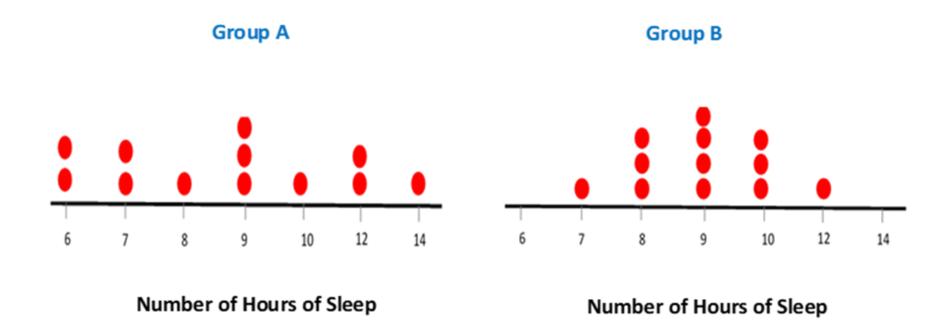
This means that on the average, the number of minutes students walk home from school differs by 1.67 minutes from the mean of 7 minutes.

## How do we compute for the Mean Absolute Deviation?

To help us solve for the Mean Absolute Deviation of a given set of data (especially if the mean is unknown), look at the sample problem below and the steps how to do it.

The dot plots on the next slide show the number of hours students sleep during weekends. The data were taken for two different groups of students.

## How do we compute for the Mean Absolute Deviation?



## How do we compute for the Mean Absolute Deviation?

Step 1: Solve for the mean for each set of data. You may round the mean to a whole number to make it easier.

Group A		
6, 6, 7, 7, 8, 9,		
9, 9, 10, 12, 12, 14		
sum of all data		
$mean = \frac{1}{number\ of\ observations}$		
109		
$mean = \frac{12}{12}$		
$mean \approx 9.08$		
mean ≈ 9		

```
Group B
  7, 8, 8, 8, 9, 9
9, 9, 10, 10, 10, 12
       sum of all data
  number of observations
    mean \approx 9.08
      mean \approx 9
```

## How do we compute for the Mean Absolute Deviation?

Step 2: Organize each data set on a table. This will make it a bit easier.

To get the distance from the mean, find the difference between each data value and the mean.

The absolute deviation or distance is **ALWAYS POSITIVE!** 

## How do we compute for the Mean Absolute Deviation?

Group A			
Number	Distance from	Absolute	
of Hours	the Mean	Deviation	
6	9 - 6 = 3	3	
6	9 - 6 = 3	3	
7	9 - 7 = 2	2	
7	9 - 7 = 2	2	
8	9 - 8 = 1	1	
9	9 - 9 = 0	0	
9	9 - 9 = 0	0	
9	9 - 9 = 0	0	
10	9 - 10 = -1	1	
12	9 - 12 = -3	3	
12	9 - 12 = -3	3	
14	9 - 14 = -5	5	

	Group B	
Number	Distance from	Absolute
of Hours	the Mean	Deviation
7	9 - 7 = 2	2
8	9 - 8 = 1	1
8	9 - 8 = 1	1
8	9 - 8 = 1	1
9	9 - 9 = 0	0
9	9 - 9 = 0	0
9	9 - 9 = 0	0
9	9 - 9 = 0	0
10	9 - 10 = -1	1
10	9 - 10 = -1	1
10	9 - 10 = -1	1
12	9 - 12 = -3	3

## How do we compute for the Mean Absolute Deviation?

### **Step 3:** Find the sum of the absolute deviations.

Group A		
Number	Distance from	Absolute
of Hours	the Mean	Deviation
6	9 - 6 = 3	3
6	9 - 6 = 3	3
7	9 - 7 = 2	2
7	9 - 7 = 2	2
8	9 - 8 = 1	1
9	9 - 9 = 0	0
9	9 - 9 = 0	0
9	9 - 9 = 0	0
10	9 - 10 = -1	1
12	9 - 12 = -3	3
12	9 - 12 = -3	3
14	9 - 14 = -5	5
	Total	23

Group B		
Number	Distance from	Absolute
of Hours	the Mean	Deviation
7	9 - 7 = 2	2
8	9 - 8 = 1	1
8	9 - 8 = 1	1
8	9 - 8 = 1	1
9	9 - 9 = 0	0
9	9 - 9 = 0	0
9	9 - 9 = 0	0
9	9 - 9 = 0	0
10	9 - 10 = -1	1
10	9 - 10 = -1	1
10	9 - 10 = -1	1
12	9 - 12 = -3	3
	Total	11

# How do we compute for the Mean Absolute Deviation?

Step 4: To get the MAD, divide the sum of the absolute deviations by the number of observations.

Group A		
Sum of the absolute	23	
deviations		
Number of Observations	12	

$$MAD = \frac{sum\ of\ the\ absolute\ deviations}{number\ o\ observations}$$
 
$$MAD = \frac{23}{12}$$
 
$$MAD \approx 1.92$$

This means that on the average, the number of hours students sleep on weekends differs by 1.92 minutes from the mean of 9 minutes.

Group B		
Sum of the absolute	11	
deviations		
Number of Observations	12	

$$MAD = \frac{sum\ of\ the\ absolute\ deviations}{number\ o\ observations}$$
 
$$MAD = \frac{11}{12}$$
 
$$MAD \approx 0.08$$

This means that on the average, the number of hours students sleep on weekends differs by 0.08 minutes from the mean of 9 minutes.

# Analyzing the Computed MAD

The value of the MAD for Group B (0.88) is lesser than that of Group A. This tells us that the data distribution has very little variability. Also, the mean is an accurate indicator of a typical value in a distribution.

The value of the MAD for Group A (1.92) is greater than that of Group B. This tells us that the data values are spread out and are far away from the mean. Also, the mean is not a good indicator of a typical value in a distribution.

Sample Problem 5: Using the mean you solved in Sample Problem 3, solve for the MAD for each set of data. Round your answers to the nearest hundredths (if needed).

(See the next slides.)

# Sample Problem 5:

Basketball Points	Exam Scores
41, 32, 45, 29, 30, 27	92, 96, 94, 88, 88, 92, 87
Mean = ?	Mean = ?
Table:	Table:
MAD = ?	MAD = ?

# Sample Problem 5:

Hours of Sleep	Number of Emails
10, 9, 13, 10, 12, 10, 8, 8, 10	15, 19, 19, 17, 18, 17, 16, 17, 15
Mean = ?	Mean = ?
Table:	Table:
MAD = ?	MAD = ?
M// 0 - F	mno - ;

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## Sample Problem 5: Solution

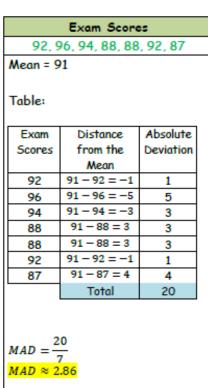
Basketball Points		
41, 32, 45, 29, 30, 27		
Mean = 34		

Table:

Basket	Distance from	Absolute
Ball	the Mean	Deviation
Points		
41	34 - 41 = -7	7
32	34 - 32 = 2	2
45	34 - 45 = -11	11
29	34 - 29 = 5	5
30	34 - 30 = 4	4
27	34 - 27 = 7	7
	Total	36

$$MAD = \frac{36}{6}$$

$$MAD = 6$$





# Sample Problem 5: Solution

Hours of Sleep
10, 9, 13, 10, 12, 10, 8, 8, 10
Mean = 10

Table:

Number	Distance	Absolute
of	from the	Deviation
Hours	Mean	
10	10 - 10 = 0	0
9	10 - 9 = 1	1
13	10 - 13 = -3	3
10	10 - 10 = 0	0
12	10 - 12 = -2	2
10	10 - 10 = 0	0
8	10 - 8 = 2	2
8	10 - 8 = 2	2
10	10 - 10 = 0	0
	Total	10
		•

$$MAD = \frac{10}{9}$$

$$MAD \approx 1.11$$

N	umber of Emails
15, 19, 1	9, 17, 18, 17, 16, 17, 15
Mean = 17	,
Table:	

Number	Distance	Absolute
of	from the	Deviation
Emails	Mean	
15	17 - 15 = 2	2
19	17 - 19 = -2	2
19	17 - 19 = -2	2
17	17 - 17 = 0	0
18	17 - 18 = -1	1
17	17 - 17 = 0	0
16	17 - 16 = 1	1
17	17 - 17 = 0	0
15	17 - 15 = 2	2
	Total	10
	<u> </u>	

$$MAD = \frac{10}{9}$$
$$MAD \approx 1.11$$