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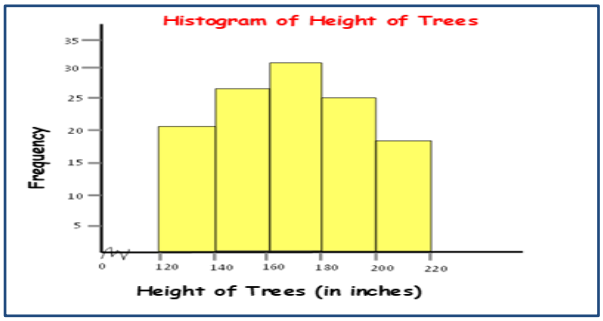
The Mean and Mean Absolute Deviation Guide Notes Math 6

The Center of Data Distribution

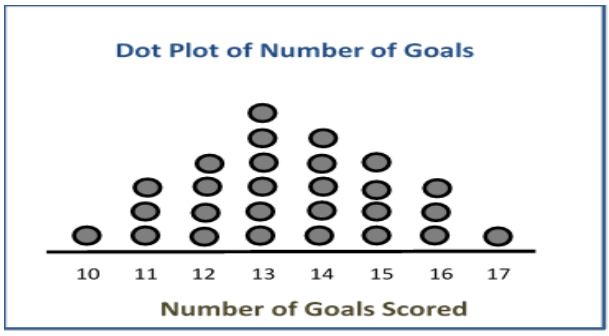
In the past lessons, you were asked to **describe the center of the data distribution** given **dot plots** and **histograms**. We only determined the middle most value of a data distribution to describe the center of the distribution. This lesson will focus on the in-depth explanation of the concept of "**center**" especially in data distributions.

The Mean as a Measure of Central Tendency

The "**mean**" or "**average**" (in simple terms) is the most appropriate way to describe and summarize data distributions that are **approximately symmetric**. Here we are trying to find that **single number** that **represents the entire set of data**.



There are other ways to find the **center** of the distribution though; you'll learn more of these in the next lessons. In the meantime, let's concentrate in finding the "mean".



Before jumping into the "**calculation**" part, let's determine the mean by using "**Fair Share**".

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The Mean and Mean Absolute Deviation Guide Notes Math 6

Interpreting the Mean as Fair Share

What does "fair share" mean?

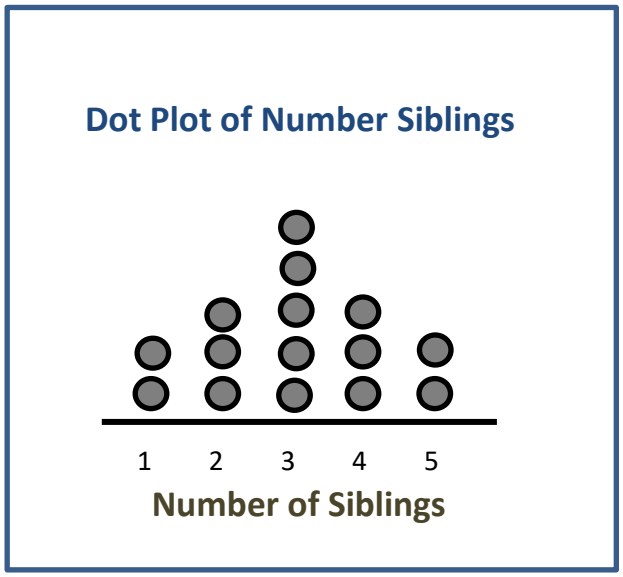
Those that have the most, give something to those with the least; until everyone has exactly the same amount.

Look at the dot plot below and let's see how "**fair share**" is done.

Sheena wants to know the typical number of siblings her five friends have.

Below are the data she collected and on the right is the dot plot that displays these data.

4, 4, 3, 1, 5, 3, 3, 1,
3, 2, 5, 2, 4, 2, 3



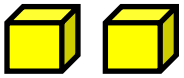
Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes **Math 6**

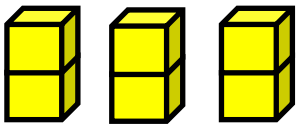
Let's stack some cubes to show how "fair share" is done.

Remember the data: 4, 4, 3, 1, 5, 3, 3, 1, 3, 2, 5, 2, 4, 2, 3

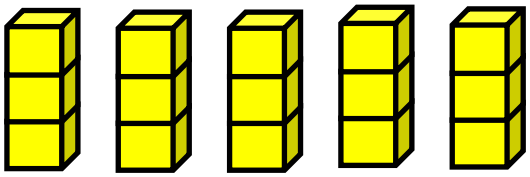
Two students have **1 sibling**:



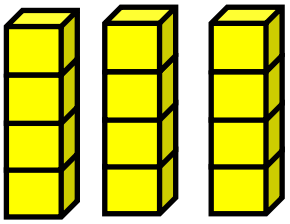
Three students have **2 siblings**:



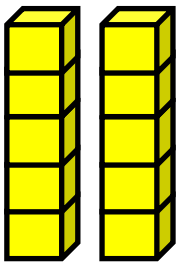
Five students have **3 siblings**:

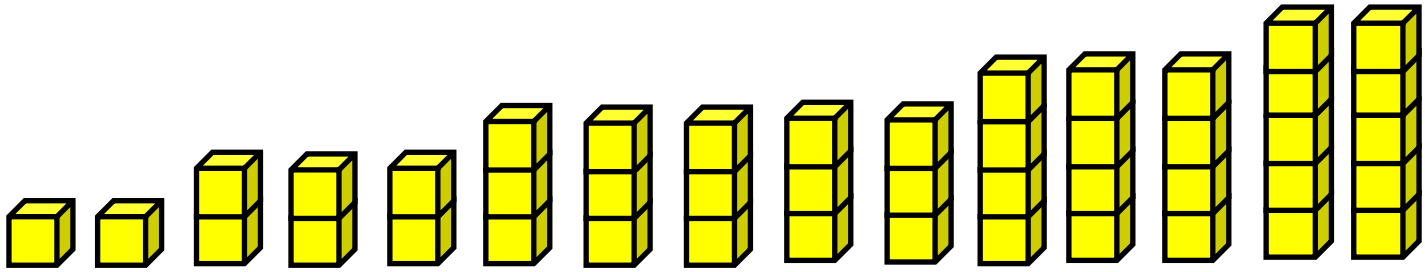


Three students have **4 siblings**:



Two students have **5 siblings**:



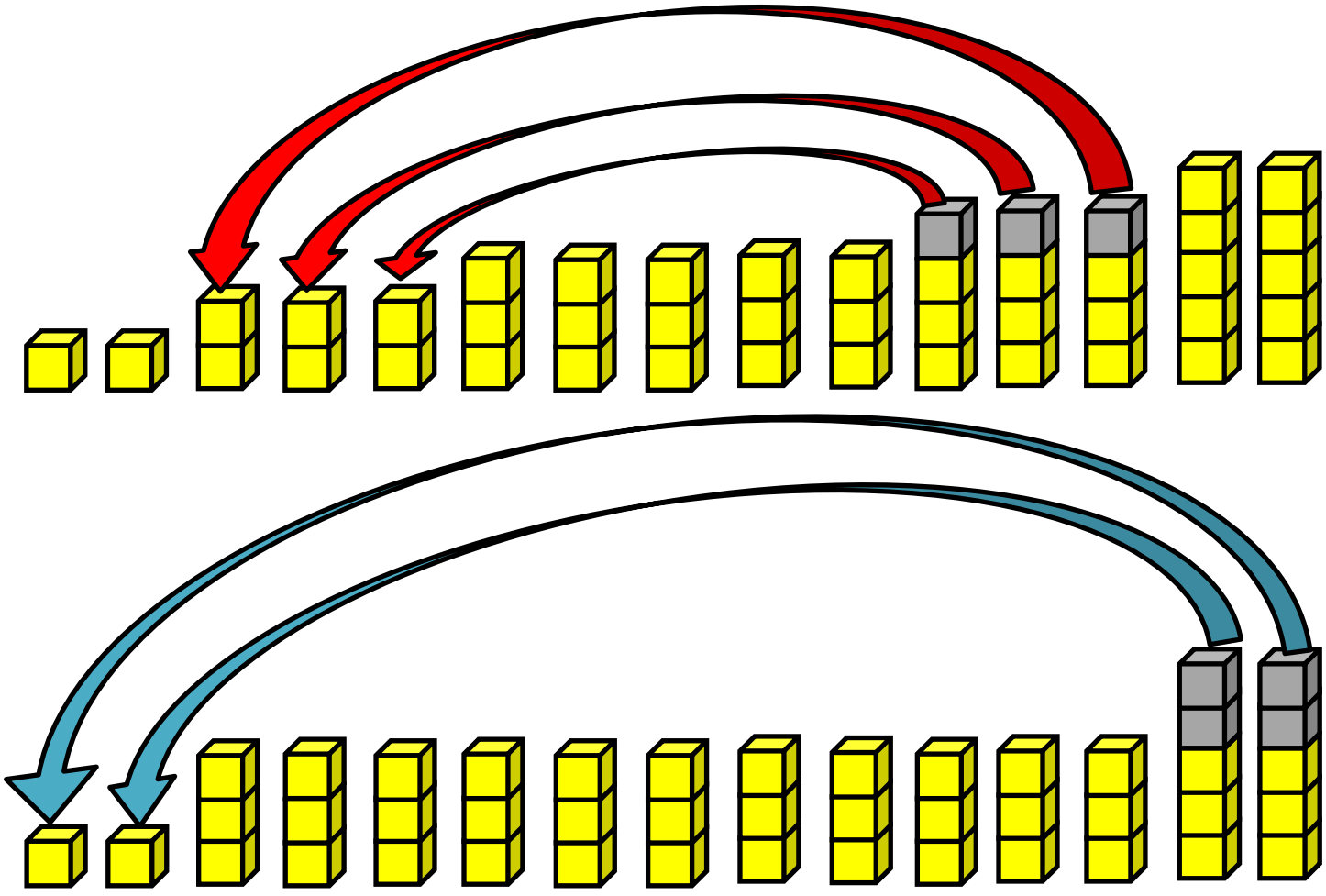


Name: _____ Period: _____ Date: _____

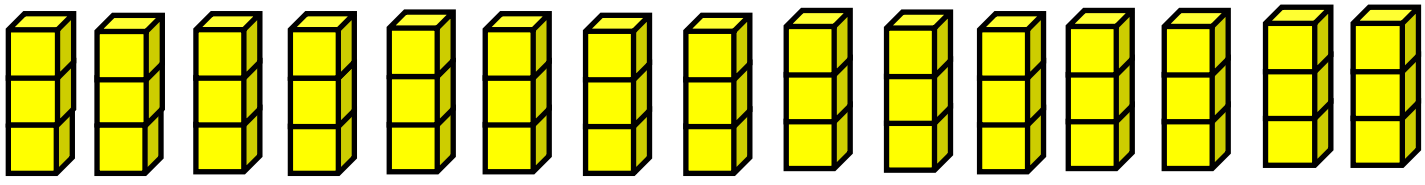
The Mean and Mean Absolute Deviation Guide Notes **Math 6**

Now, it's time to share!

Those that have the most, give something to those with the least; until everyone has exactly the same amount.



Now, everyone has exactly the same number of cubes.



Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

Sheena wants to know the typical number siblings her five friends have.

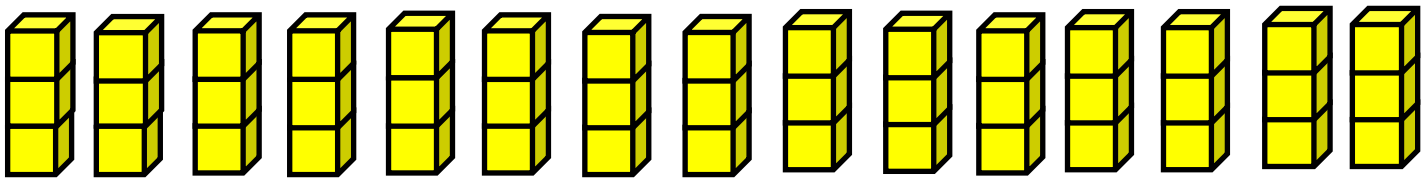
Below are the data she collected

4, 4, 3, 1, 5, 3, 3, 1,
3, 2, 5, 2, 4, 2, 3

This is how we interpret the mean as "fair share". Now each one has 3 cubes. This means that **the center of the distribution** is **3**.

This means that the mean or average number of siblings for each student is 3. Also, 3 is the single number that represents the given set of data.

The typical number of siblings Sheena's friends have is 3.



Let's have another example of interpreting the mean as "fair Share".

Tom wants to know his mean score in his five Statistics tests.

Below are his scores.

88, 86, 94, 92, 90

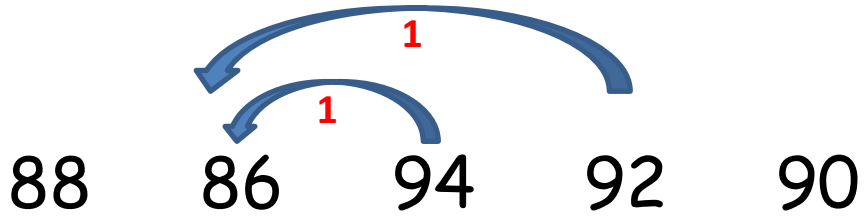
Interpret the mean as "fair share".

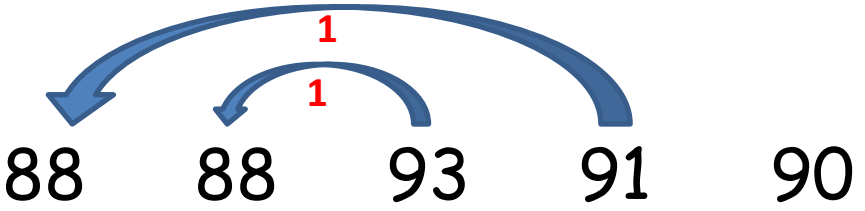
Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes **Math 6**

Remember:

Those that have the most, give something to those with the least; until everyone has exactly the same amount.





90 90 90 90 90

Therefore, Tom's mean score in his 5 Statistics tests is **90**.

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

Sample Problem 1: Read the problem and interpret the mean as fair share.

1. Liza wants to know the typical number of text messages five of her friends send in a day.

Below are the data she collected

4, 3, 6, 8, 4

Using cubes, interpret the mean as "fair share".

Solution:

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

2. Stanley wants to know the typical amount of milk (in Liters) his 5 cows produce in a day.

Below are the data he collected:

24, 27, 30, 26, 23

Interpret the mean as "fair share".

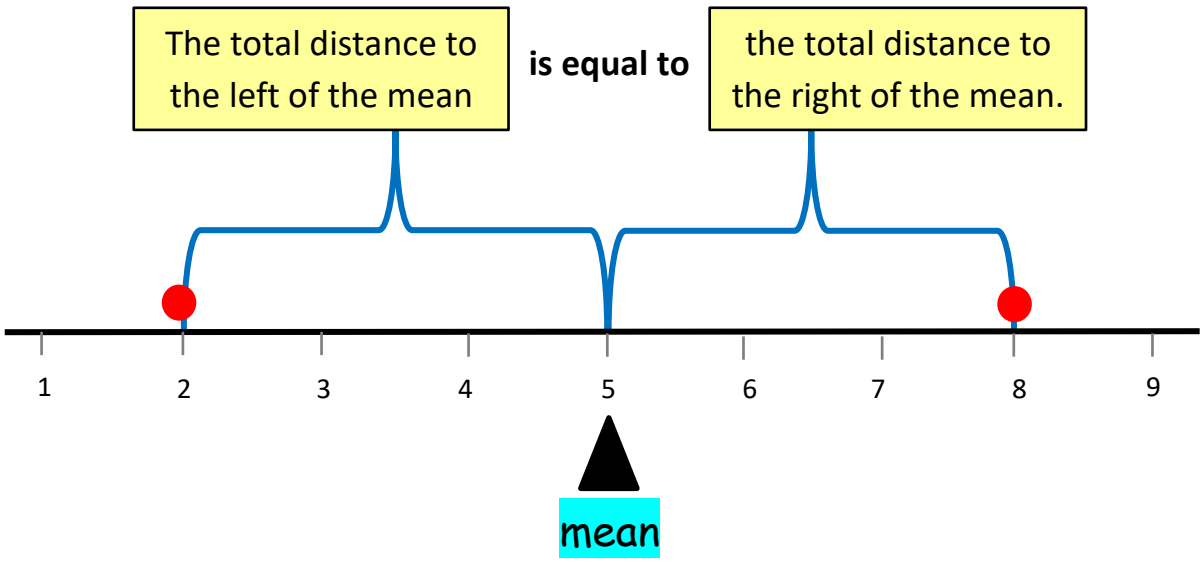
Solution:

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

Mean as a Balancing Point

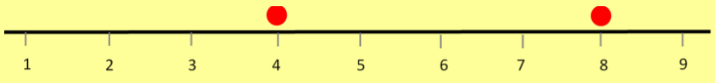
To interpret the mean as a balancing point, we need to understand that the following distances from the mean are equal:



Look at the example below and find the mean.

Below is the dot plot that shows the number of minutes it takes for two students to walk home from school.

Dot Plot of Number of Minutes



Number of Minutes

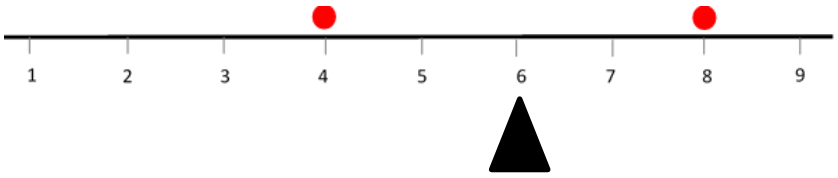
Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

The questions below will serve as a guide to determine the mean:

1. Where should the balance point be?

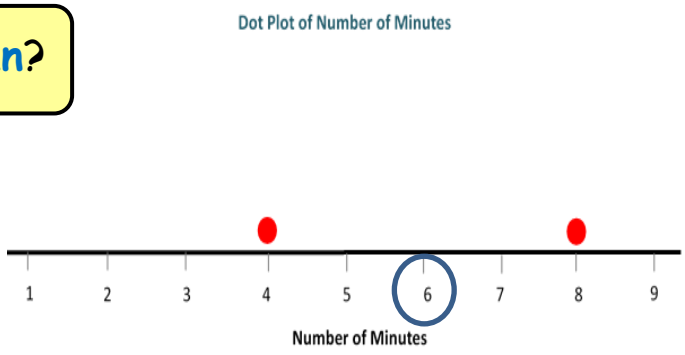
Remember that the balance point represents the mean of the data. Also, the total distances to the left of the balancing point must be equal to the total distances to its right. In the dot plot above, the balancing point must be at **6**.



The distance to the left of the balancing point, between 3 and 6 is **3**.

The distance to the right of the balancing point, between 3 and 8 is **3**.

2. What is the mean?



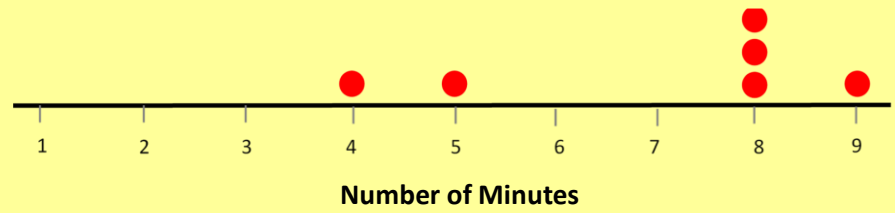
The mean is **6**.

Name: _____ Period: _____ Date: _____

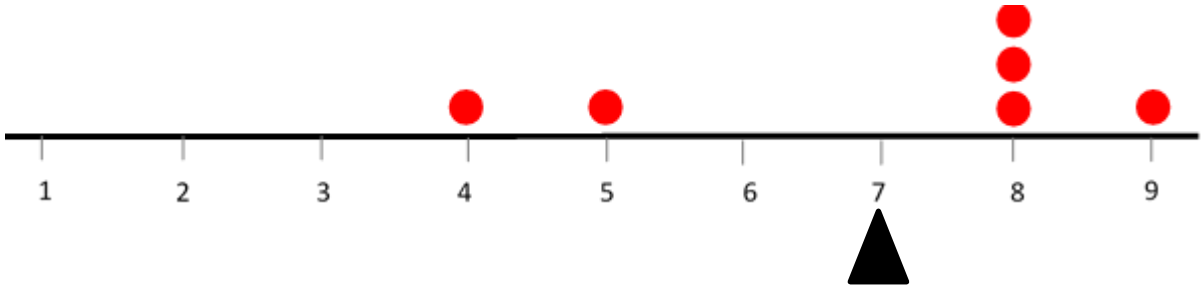
The Mean and Mean Absolute Deviation Guide Notes **Math 6**

If we add more data to the problem, will the mean change?

Below is the dot plot that shows the number of minutes it takes for six students to walk home from school.



To have equal total distances to the left and to the right of the mean, the balance point must be at **7**. Therefore, the mean must be **7**.



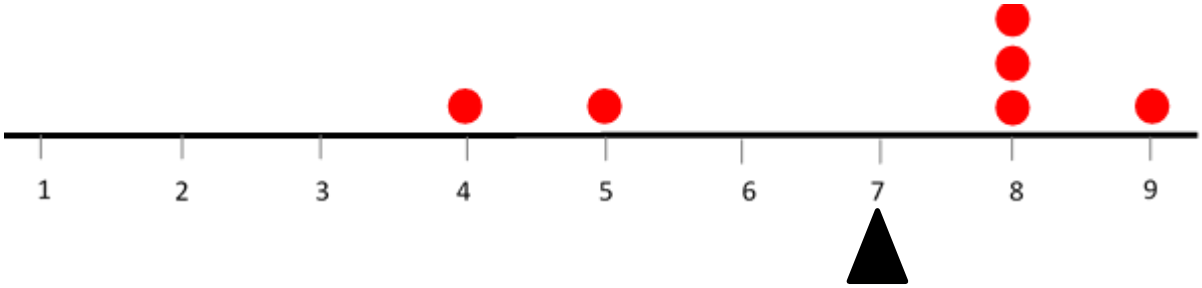
One of the distances to the left of the mean (between 5 and 7) is 2.

One of the distances to the left of the mean (between 4 and 7) is 3.

Total distance to the left of the mean: $2 + 3 = \mathbf{5}$

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6



One of the distances to the right of the mean (between 7 and 8) is 1. Since there are three data point at 8, we'll count the distance between 7 and 8 three times: $\mathbf{1 + 1 + 1 = 3}$

One of the distances to the right of the mean (between 7 and 9) is **2**.

Total distance to the right of the mean: $3 + 2 = \mathbf{5}$

Now, we can really say that the value that represents the typical number of minutes six students walk home from school is 7. This means that the mean of the given data is **7**.

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

Sample Problem 2: Display the data using a dot plot, find the balancing point to determine the mean.

Sheena wants to know the typical number of pets her twelve friends have.

Below are the data she collected:

4, 1, 2, 4, 2, 4, 2, 4, 3, 4,

Solution:

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

Calculating the Mean

Aside from using "fair share" and "balancing point" to determine the mean of a given set of data, this too can be done mathematically by calculating it using a formula.

Formula for the mean:

$$mean = \frac{sum\ of\ all\ data}{number\ of\ observations}$$

Let's use the previous examples to check if the answers will remain the same. To get the mean of the data below, the concept of "fair share" was used.

Sheena wants to know the typical number of siblings her five friends have.

Below are the data she collected:

4, 4, 3, 1, 5, 3, 3, 1, 3, 2, 5, 2, 4, 2, 3

We used the "**fair share**" method to determine the mean.

Here, the **mean** is **3**.

Will the mean be the same if we calculate it using the formula?

Let's find it out!

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

Sheena wants to know the typical number of siblings her five friends have.

Below are the data she collected:

4, 4, 3, 1, 5, 3, 3, 1, 3, 2, 5, 2, 4, 2, 3

$$mean = \frac{sum\ of\ all\ data}{number\ of\ observations}$$

Sum of all data:
Add up all the data collected.

Number of observations:
Count the number of data you have.

$$mean = \frac{sum\ of\ all\ data}{number\ of\ observations}$$

$$mean = \frac{4+4+3+1+5+3+3+1+3+2+5+2+4+2+3}{15}$$

$$mean = \frac{45}{15}$$

$$mean = 3$$

So it is **TRUE**! The **fair share** method and the **formula** gave us **the same** result!

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

Let's try this one too!

Tom wants to know his mean score in his five Statistics tests.

Below are his scores.

88, 86, 94, 92, 90

Using the "**fair share**" method the **mean** is **90**.

Will the mean be the same if we calculate it using the formula?

Let's find it out!

$$mean = \frac{sum\ of\ all\ data}{number\ of\ observations}$$

$$mean = \frac{88 + 86 + 94 + 92 + 90}{5}$$

$$mean = \frac{450}{5}$$

$$mean = 90$$

The methods may be different, but the results are still the same.
The mean is **90**.

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

Sample Problem 3: Find the mean for each set of data using the formula.

Basketball Points
41, 32, 45, 29, 30, 27
Mean = ?

Exam Scores
92, 96, 94, 88, 88, 92, 87
Mean = ?

Hours of Sleep
10, 9, 13, 10, 12, 10, 8, 8, 10
Mean = ?

Number of Emails
15, 19, 19, 17, 18, 17, 16, 17, 15
Mean = ?

Body Length (in cm)
142.5, 137.25, 150.75, 139.5
Mean = ?

Height of Students (in inches)
57, 59, 56, 59, 62, 60, 58, 59, 57
Mean = ?

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

The Variability in the Distribution

Variability in a distribution refers to how "**spread out**" or "**scattered**" the data around the mean. Sometimes, distributions may have the same mean but can have different variability. This measures how much the data differ from each other.

There are two things you need to look out for:

1. **Are the data spread out around the mean?**
   In this case, there is a **greater variability** (wide spread) in the distribution. Thus, the **mean is not a good representation** of a typical value in a data set.

2. **Are the data clustered around the mean?**
   In this case, there is a **lesser variability** (closer to the mean) in the distribution. Thus, the mean indicates an **accurate representation** of a typical value in a data set.

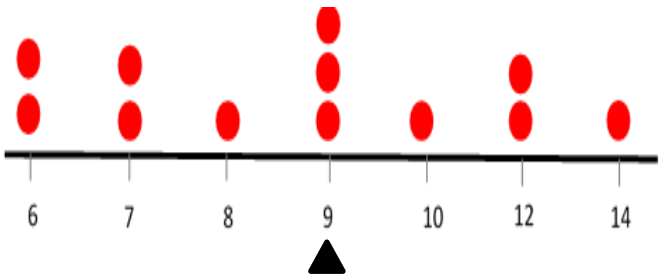
Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

The dot plots below show the number of hours students sleep during weekends. The data were taken for two different groups of students.

Both data set has the same mean, 9.

Group A

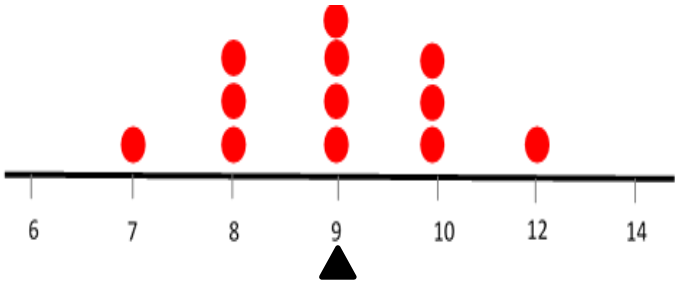


Number of Hours of Sleep

The data in Group A ranges from 6 hours to 14 hours. This shows a greater variability because they are spread out around the mean.

Thus, its mean which is 9 is not a good indicator of a typical number of hours students sleep on weekends.

Group B



Number of Hours of Sleep

The data in Group B ranges from 7 hours to 12 hours. This shows a lesser variability because they are clustered around the mean.

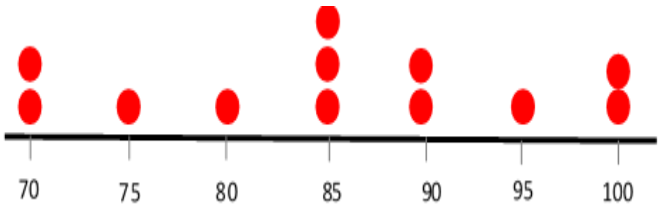
Thus, its mean which is 9 is an accurate indicator of a typical number of hours students sleep on weekends.

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

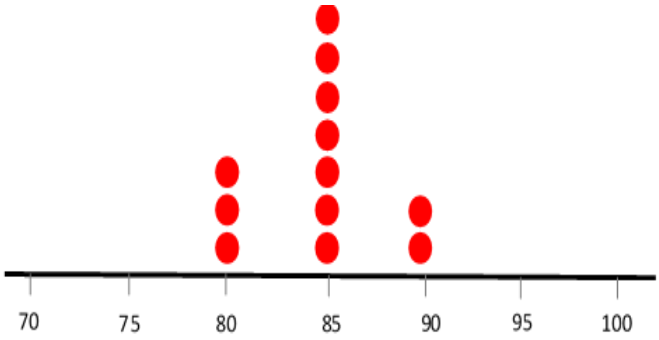
Sample Problem 4: Below are the dot plots of the scores in a Math test from two different groups. Analyze the dot plots and answer the questions that follow.

Group A



Test Scores

Group B



Test Scores

Questions:

1. What is the mean score for each group? Compute for the mean score. (Round off to a whole number if needed)

2. Which distribution has the mean that is a more accurate indicator of the typical test score?

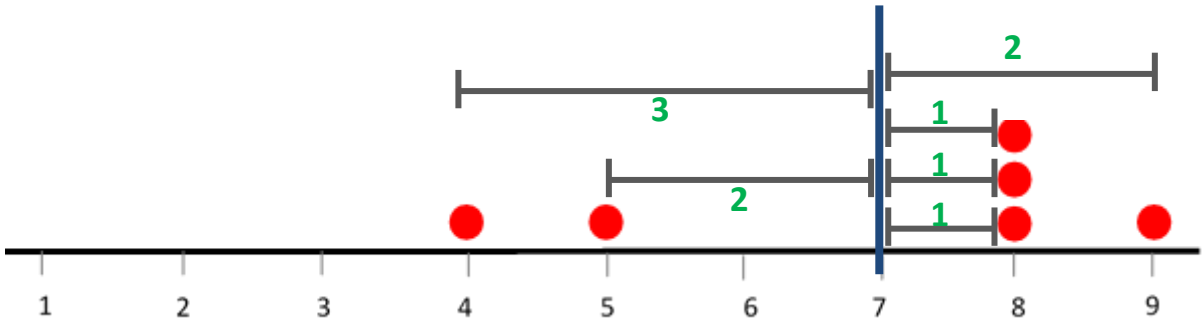
Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

The Mean Absolute Deviation

Before we discuss the mean absolute deviation, lets first understand what "absolute deviation" means.

Absolute deviation is the distance of a data value form the mean. To make it even simpler, it determines how far a data value is form the mean. Below is the dot plot that shows the number of minutes it takes for six students to walk home from school. Here, the mean is **7**.



Number of Minutes	Deviation from the Mean (Distance and Direction)	Absolute Deviation (Distance form the Mean)
4	*3 to the left*	3
5	*2 to the left*	*2*
8	*1 to the right*	*1*
8	*1 to the right*	*1*
8	*1 to the right*	*1*
9	*2 to the right*	*2*

The total distances to the left of the mean is equal to the total distances to the to its right.

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

The **Mean Absolute Deviation (MAD)** is the average of the absolute deviations, all the distances of the given data form the mean. Here's what the Mean Absolute Deviation tell us about the variability of a distribution.

1. The value of the MAD tells us about average distance of the data values from the mean.

2. A smaller value of MAD tells us that the data distribution has very little variability. Also, the mean is an accurate indicator of a typical value in a distribution.

3. A larger value of MAD tells us that the data values are spread out and are far away from the mean. Also, the mean is not a good indicator of a typical value in a distribution.

To solve for the MAD for this set of data, here's what we need to do.

Number of Minutes	Deviation from the Mean (Distance and Direction)	Absolute Deviation (Distance form the Mean)
4	*3 to the left*	3
5	*2 to the left*	*2*
8	*1 to the right*	*1*
8	*1 to the right*	*1*
8	*1 to the right*	*1*
9	*2 to the right*	*2*

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

This will be easier because we already know the mean. Find the sum of the absolute deviation:

Number of Minutes	Deviation from the Mean (Distance and Direction)	Absolute Deviation (Distance form the Mean)
4	*3 to the left*	3
5	*2 to the left*	*2*
8	*1 to the right*	*1*
8	*1 to the right*	*1*
8	*1 to the right*	*1*
9	*2 to the right*	*2*
		Total = 10

To get the MAD, divide the sum of the absolute deviations and by the number of observations.

$$MAD = \frac{sum\ of\ the\ absolute\ deviations}{number\ of\ observations}$$

$$MAD = \frac{10}{6}$$

$$MAD \approx 1.67$$

This means that on the average, the number of minutes students walk home from school differs by 1.67 minutes from the mean of 7 minutes.

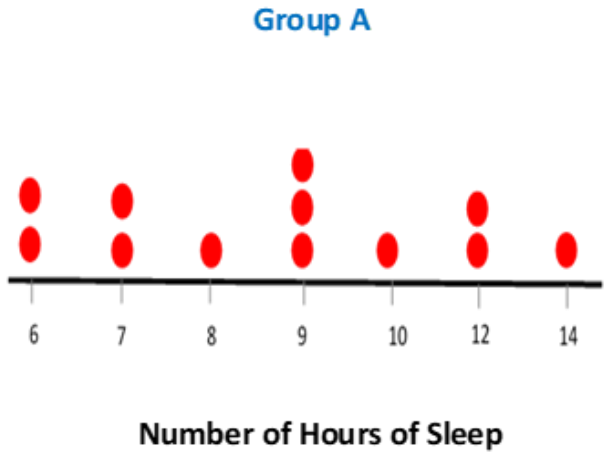
Name: _____ Period: _____ Date: _____

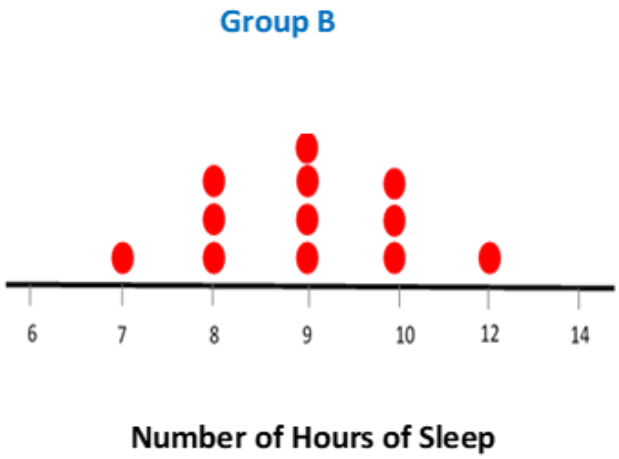
The Mean and Mean Absolute Deviation Guide Notes Math 6

How do we compute for the Mean Absolute Deviation?

To help us solve for the Mean Absolute Deviation of a given set of data (especially if the mean is unknown), look at the sample problem below and the steps how to do it.

The dot plots below show the number of hours students sleep during weekends. The data were taken for two different groups of students.





Step 1: Solve for the mean for each set of data. You may round the mean to a whole number to make it easier.

Group A	Group B
6, 6, 7, 7, 8, 9, 9, 9, 10, 12, 12, 14	7, 8, 8, 8, 9, 9 9, 9, 10, 10, 10, 12
$mean = \frac{sum\ of\ all\ data}{number\ of\ observations}$ $mean = \frac{109}{12}$ $mean \approx 9.08$ $mean \approx 9$	$mean = \frac{sum\ of\ all\ data}{number\ of\ observations}$ $mean = \frac{109}{12}$ $mean \approx 9.08$ $mean \approx 9$

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

Step 2: Organize each data set on a table. This will make it a bit easier.

To get the distance from the mean, **find the difference between each data value and the mean**.

The absolute deviation or distance is **ALWAYS POSITIVE**!

Group A		
Number of Hours	Distance from the Mean	Absolute Deviation
6	$9-6=3$	3
6	$9-6=3$	3
7	$9-7=2$	2
7	$9-7=2$	2
8	$9-8=1$	1
9	$9-9=0$	0
9	$9-9=0$	0
9	$9-9=0$	0
10	$9-10=-1$	1
12	$9-12=-3$	3
12	$9-12=-3$	3
14	$9-14=-5$	5

Group B		
Number of Hours	Distance from the Mean	Absolute Deviation
7	$9-7=2$	2
8	$9-8=1$	1
8	$9-8=1$	1
8	$9-8=1$	1
9	$9-9=0$	0
9	$9-9=0$	0
9	$9-9=0$	0
9	$9-9=0$	0
10	$9-10=-1$	1
10	$9-10=-1$	1
10	$9-10=-1$	1
12	$9-12=-3$	3

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

Step 3: Find the sum of the absolute deviations.

Group A		
Number of Hours	Distance from the Mean	Absolute Deviation
6	$9-6=3$	3
6	$9-6=3$	3
7	$9-7=2$	2
7	$9-7=2$	2
8	$9-8=1$	1
9	$9-9=0$	0
9	$9-9=0$	0
9	$9-9=0$	0
10	$9-10=-1$	1
12	$9-12=-3$	3
12	$9-12=-3$	3
14	$9-14=-5$	5
	Total	23

Group B		
Number of Hours	Distance from the Mean	Absolute Deviation
7	$9-7=2$	2
8	$9-8=1$	1
8	$9-8=1$	1
8	$9-8=1$	1
9	$9-9=0$	0
9	$9-9=0$	0
9	$9-9=0$	0
9	$9-9=0$	0
10	$9-10=-1$	1
10	$9-10=-1$	1
10	$9-10=-1$	1
12	$9-12=-3$	3
	Total	11

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

Step 4: To get the MAD, divide the sum of the absolute deviations by the number of observations.

Group A	
Sum of the absolute deviations	23
Number of Observations	12

$$MAD = \frac{sum\ of\ the\ absolute\ deviations}{number\ o\ observations}$$

$$MAD = \frac{23}{12}$$

$$MAD \approx 1.92$$

This means that on the average, the number of hours students sleep on weekends differs by 1.92 minutes from the mean of 9 minutes.

Group B	
Sum of the absolute deviations	11
Number of Observations	12

$$MAD = \frac{sum\ of\ the\ absolute\ deviations}{number\ o\ observations}$$

$$MAD = \frac{11}{12}$$

$$MAD \approx 0.08$$

This means that on the average, the number of hours students sleep on weekends differs by 0.08 minutes from the mean of 9 minutes.

Analyzing the Computed MAD

The value of the MAD for Group B (0.88) is lesser than that of Group A. This tells us that the data distribution has very little variability. Also, the mean is an accurate indicator of a typical value in a distribution.

The value of the MAD for Group A (1.92) is greater than that of Group B. This tells us that the data values are spread out and are far away from the mean. Also, the mean is not a good indicator of a typical value in a distribution.

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

Sample Problem 5: Using the mean you solved in **Sample Problem 3**, solve for the MAD for each set of data.

Basketball Points	Exam Scores
41, 32, 45, 29, 30, 27	92, 96, 94, 88, 88, 92, 87
Mean = ? Table: MAD = ?	Mean = ? Table: MAD = ?

Name: _____ Period: _____ Date: _____

The Mean and Mean Absolute Deviation Guide Notes Math 6

Hours of Sleep	Number of Emails
10, 9, 13, 10, 12, 10, 8, 8, 10	15, 19, 19, 17, 18, 17, 16, 17, 15
Mean = ? Table: MAD = ?	Mean = ? Table: MAD = ?