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The Area of Polygons Through Composition and Decomposition

Unit 6 Lesson 4

Math 6

Students will be able to:

Find the lengths of the unknown sides of an irregularly-shaped polygon.

Find the area of the region bounded by an irregular polygon by decomposing the polygon into triangles, rectangles or other polygons.

Find the area of other quadrilaterals by decomposing triangles or rectangles.

Understand that the area of a polygon is actually the region bounded by the polygon.

Key Vocabulary:

Decomposition

Composition

Irregularly-shaped Polygon

Polygon

Area

Trapezoid

Parallelogram

Composition and Decomposition Defined

Composition and **Decomposition** came from the root word "compose" which means to join or put together.

On the other hand, **decomposition** means to take apart or to separate.

Composition and Decomposition Defined

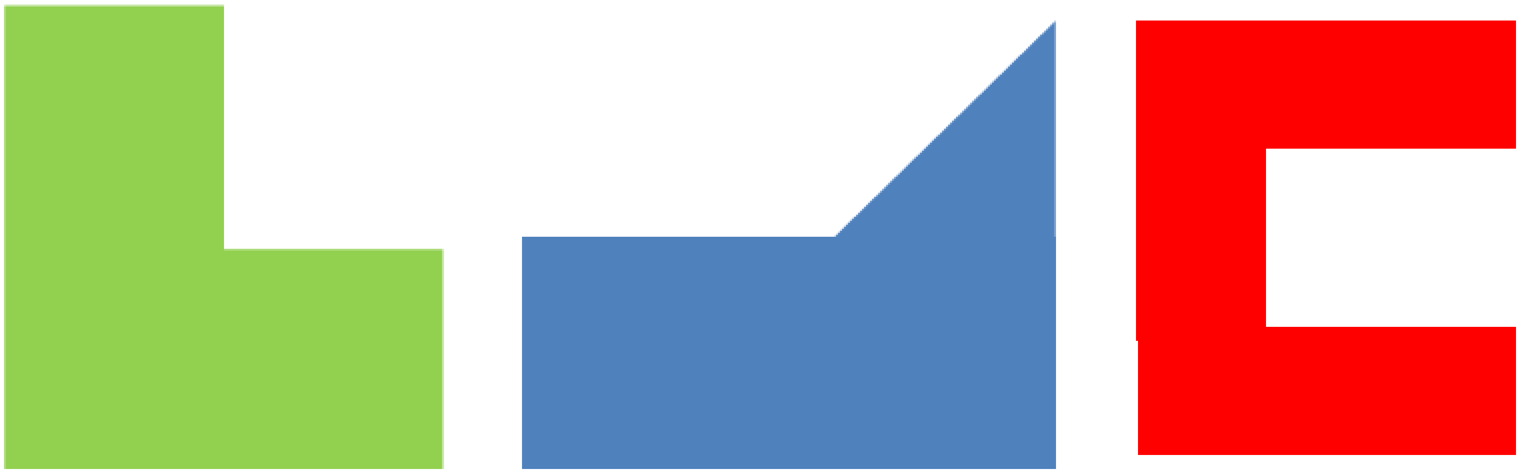
In this lesson, decomposition will be used to find the area of irregularly-shaped polygons by separating them into triangles and/or other polygons.

The area of each separated part can then be added to find the area of the entire figure.

To determine the area of an irregularly-shaped polygon, the following have to be considered:

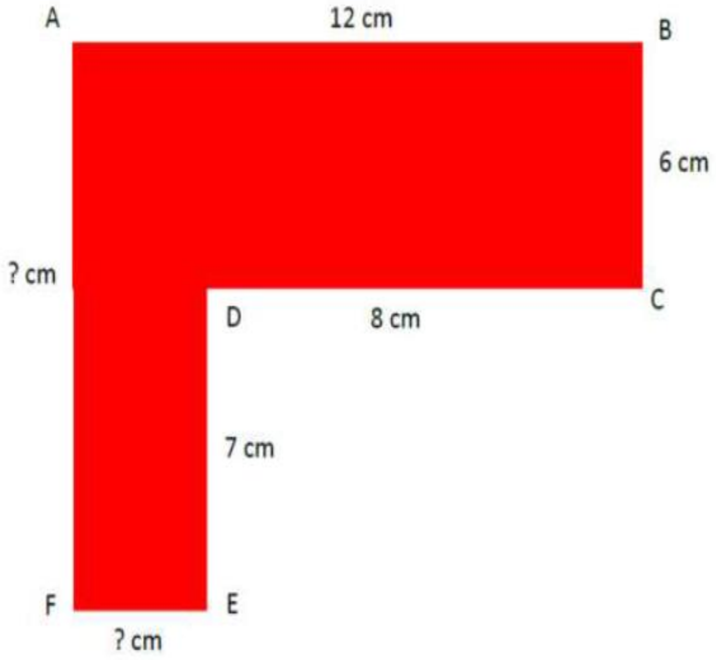
- the skill in finding the length of the unknown sides
- making decisions as to where to separate the figure to decompose irregularly-shaped polygons into rectangles and/or triangles

How is the area of irregularly-shaped polygons be determined?



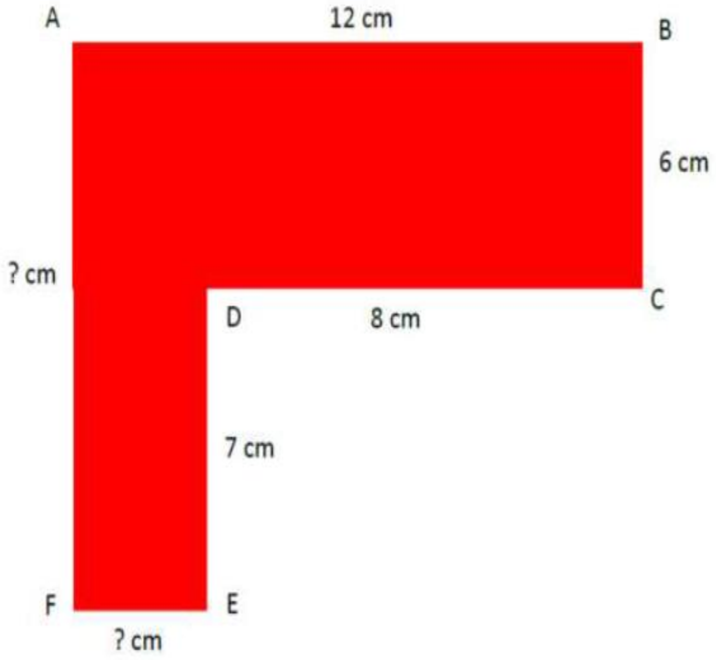
Finding the Unknown Lengths

As mentioned, one very important skill needed in finding the area of an irregularly-shaped polygon is finding the lengths of any missing or unknown side. Consider the figure:



How can the lengths of the missing sides be known?

In the figure, the lengths of sides *AF* and *FE* are unknown. To visually show the progression of the figures, vertical and/or horizontal lines will be drawn to separate the figure into rectangles.



To find the missing length, we either add or subtract.

When do we add?

We add when the shorter sides are given (horizontal or vertical), to get the longer side.

When do we subtract?

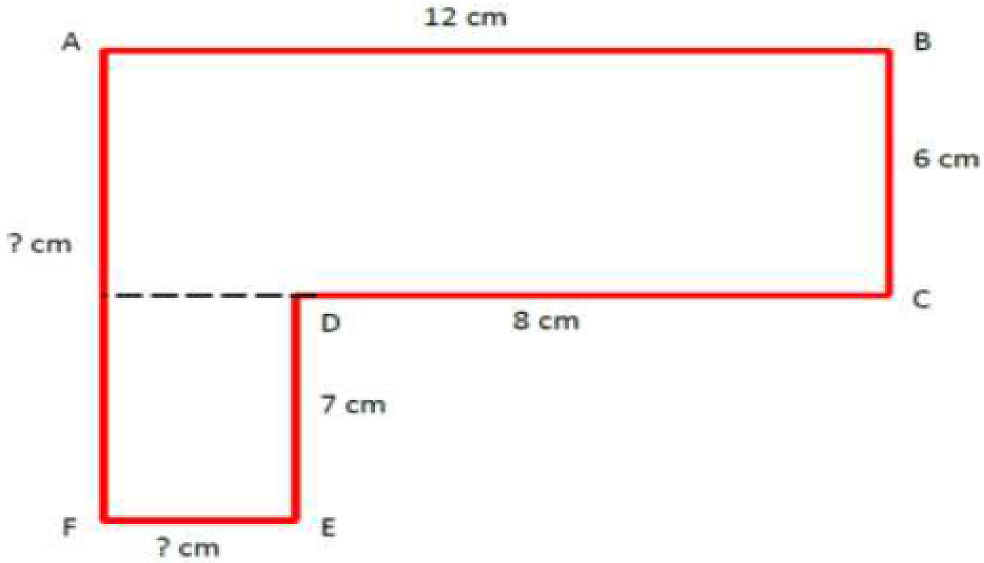
We subtract when one long side and one short side (horizontal or vertical) are given.

Decomposing Polygons into Rectangles

One's best judgment is used to decide how and where to separate an irregular polygon. Below is an example of the progression of figures.

Decomposing Polygons into Rectangles

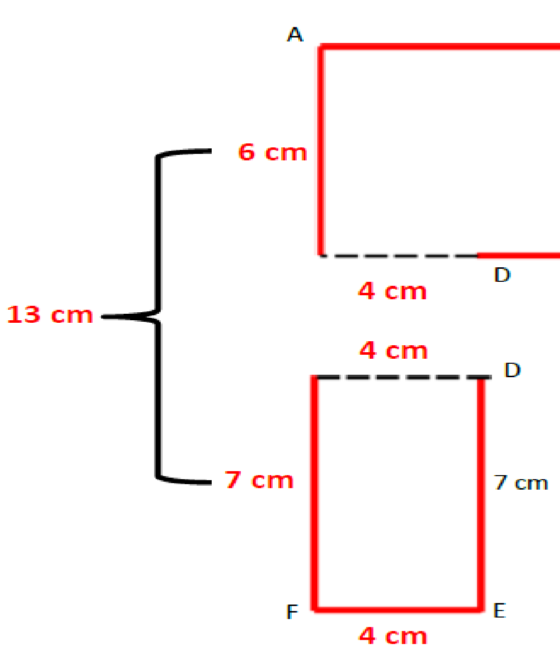
Option A



Decomposing Polygons into Rectangles

Cutting the entire figure horizontally divides the figure into two separate rectangles.

Finding the Unknown Lengths



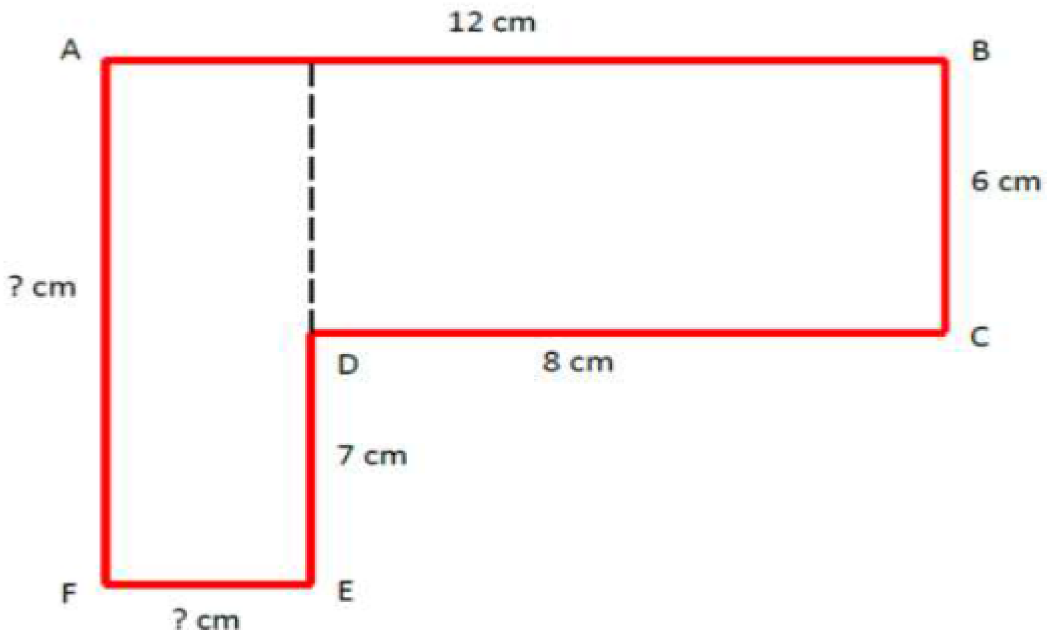
Therefore, the unknown sides have lengths:

AF = 13 cm

FE = 4 cm

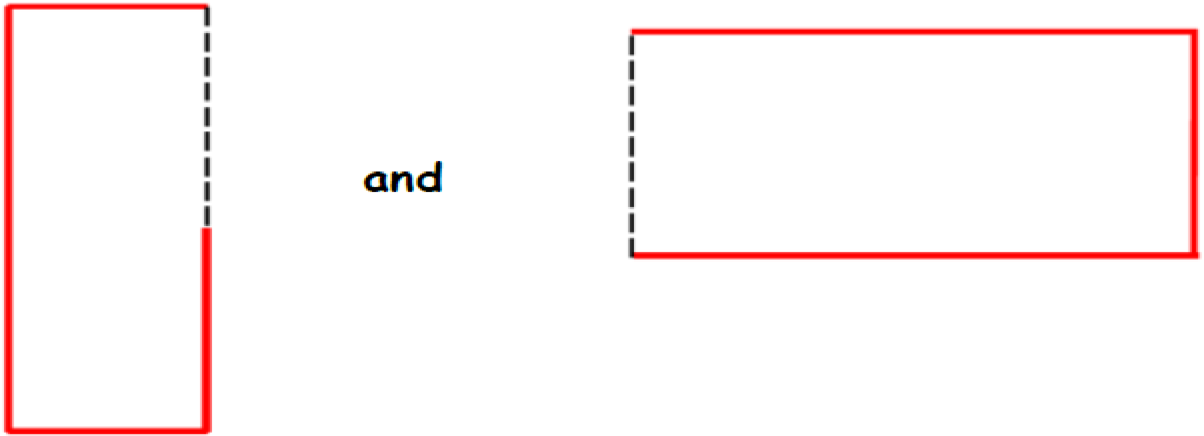
Decomposing Polygons into Rectangles

Option B

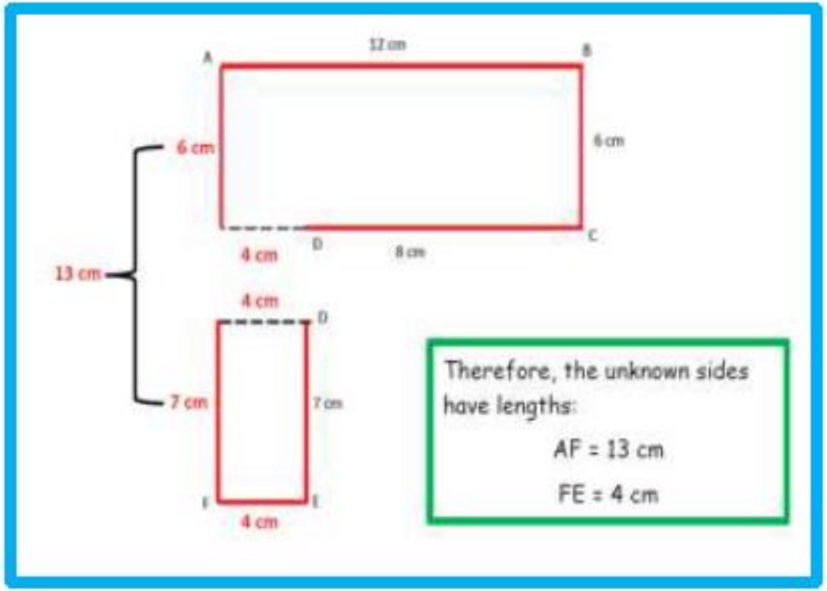


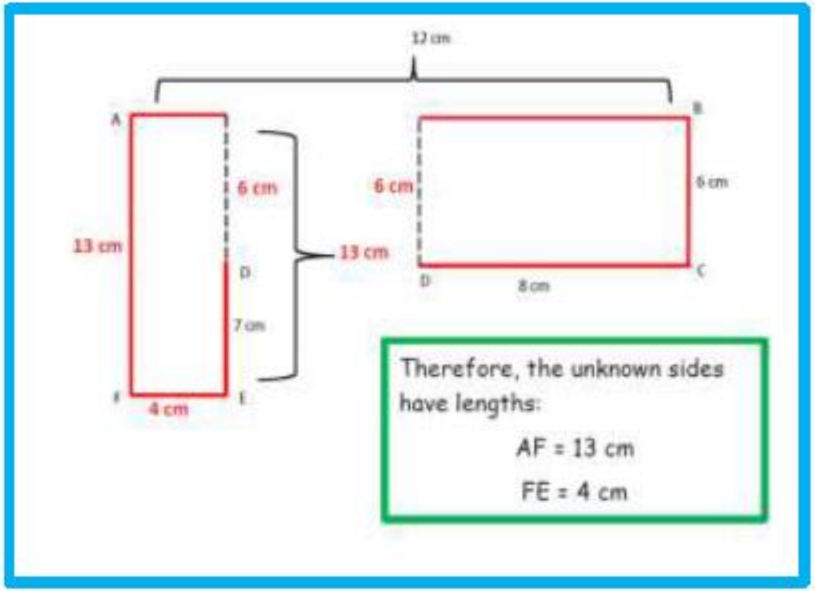
Decomposing Polygons into Rectangles

Cutting the entire figure vertically divides the figure into two separate rectangles.



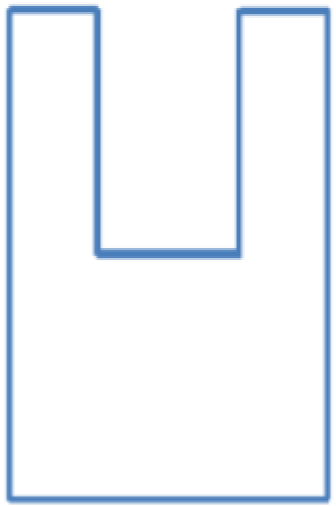
The same values will come out whether the irregular polygons are separated into rectangles, horizontally or vertically



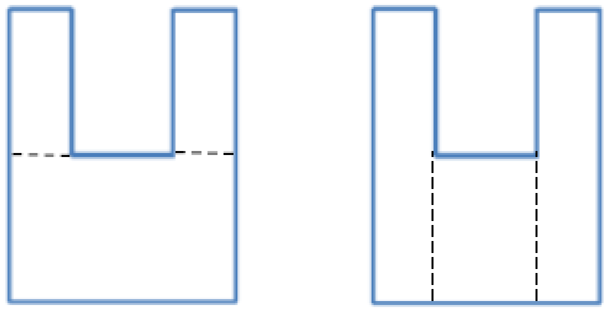


Sample Problem 1

Decompose the given figure into rectangles.

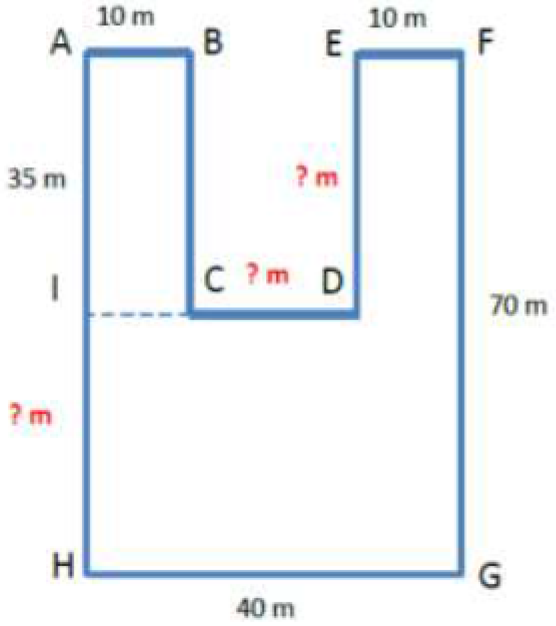


Solution: (answers may vary)



Sample Problem 2:

Find the missing lengths in the given figure.



Solution:

CD = 20 m

IH = 35 m

ED = 35 m

The Area of Irregular Polygons

Steps to find the area of an irregularly-shaped polygon:

Step 1: Decompose the irregularly-shaped polygon into rectangles.

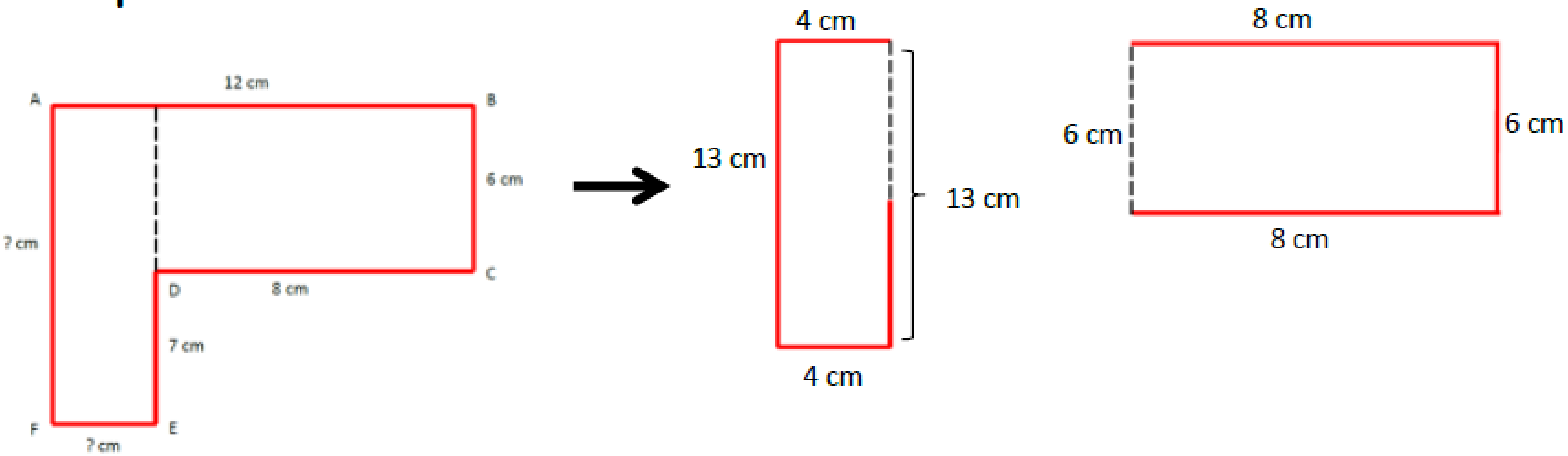
Step 2: Determine the length of any unknown side.

Step 3: Solve for the area of each the decomposed rectangles.

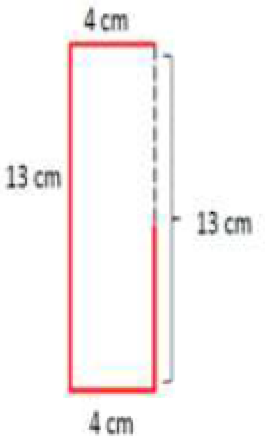
Step 4: Sum up all the area to get the area of the entire figure.

The Area of Irregular Polygons

Example:



The Area of Irregular Polygons

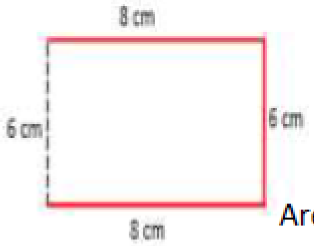


Area of a Rectangle

$A = l \times w$

$A = (4\text{cm})(13\text{cm})$

$A = 52\ \text{cm}^2$



Area of a Rectangle

$A = l \times w$

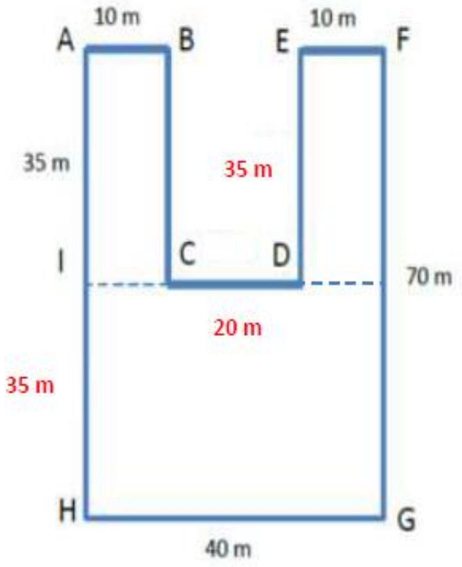
$A = (8\text{cm})(6\text{cm})$

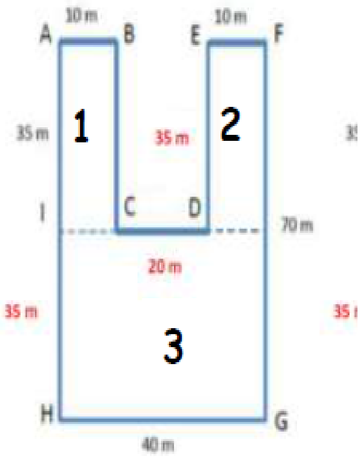
$A = 48\ \text{cm}^2$

Total area of the figure = $52\ \text{cm}^2 + 48\ \text{cm}^2 =$ **$100\ \text{cm}^2$**

Sample Problem 3:

Find the area of the figure below.





Solution:

Total Area = the sum of the area of the 3 rectangles.

Total Area = $(l_1 \times w_1) + (l_2 \times w_2) + (l_3 \times w_3)$

Total Area = $(10 \times 35) + (10 \times 35) + (40 \times 35)$

Total Area = $350 + 350 + 1400$

Total Area = $2100\ m^2$

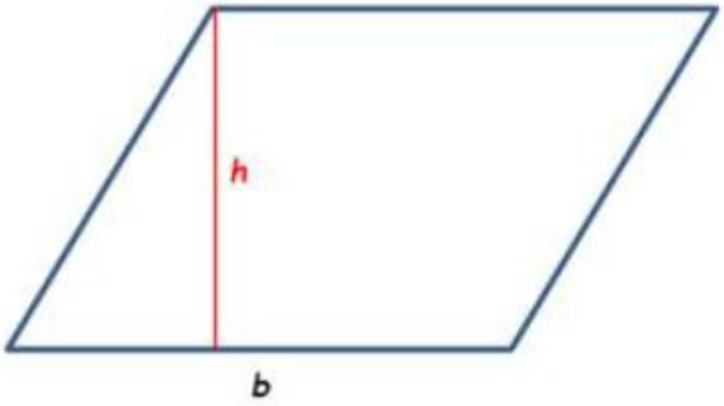
Decomposing Polygons into Rectangles and Triangles

Decomposing Parallelograms

The area of a parallelogram can be determined by multiplying any of its base "b" to its corresponding altitude/height "h".

Decomposing Parallelograms

How can the area of a parallelogram be determined using only triangles?

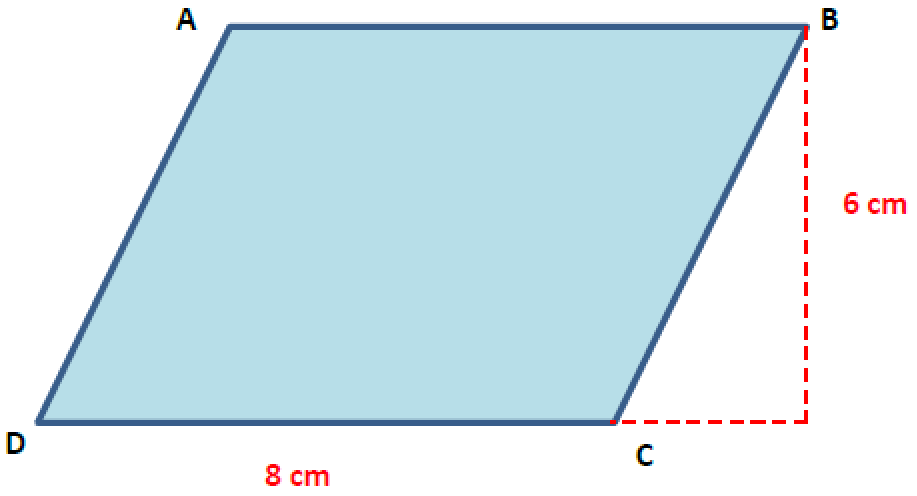


The area of a parallelogram:

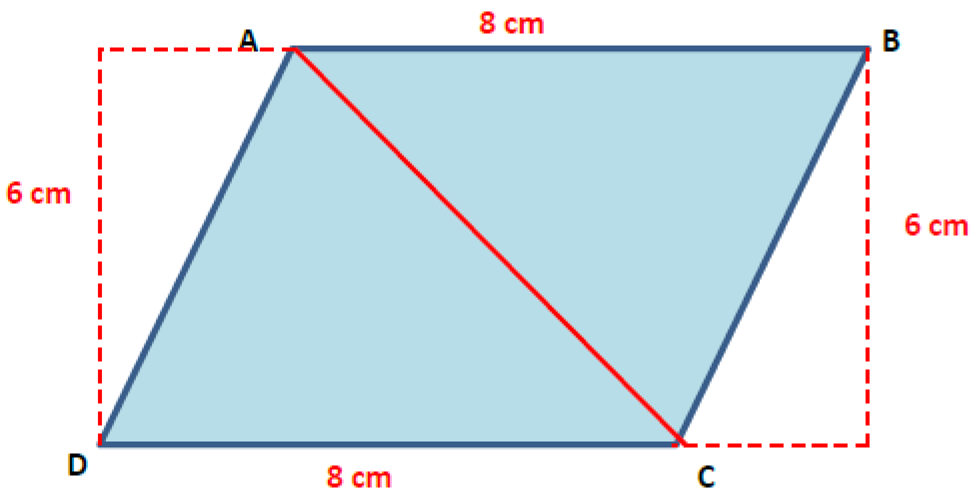
$A = b \times h$

Example:

Below is parallelogram ABCD with base 8 cm and height 6 cm.



Draw a diagonal from A to C, so 2 triangles are formed.



The 2 triangles formed are ***Triangle ABD*** and ***Triangle BCD***. Find the area of the two triangles.

Area of Triangle ABC

$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

$A = \frac{(8)(6)}{2} = \frac{48}{2} = 24\text{ cm}^2$

Area of Triangle ACD

$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

$A = \frac{(8)(6)}{2} = \frac{48}{2} = 24\text{ cm}^2$

Therefore,

Area of Parallelogram ABCD =

Area of Triangle ABD + Area of Triangle BCD

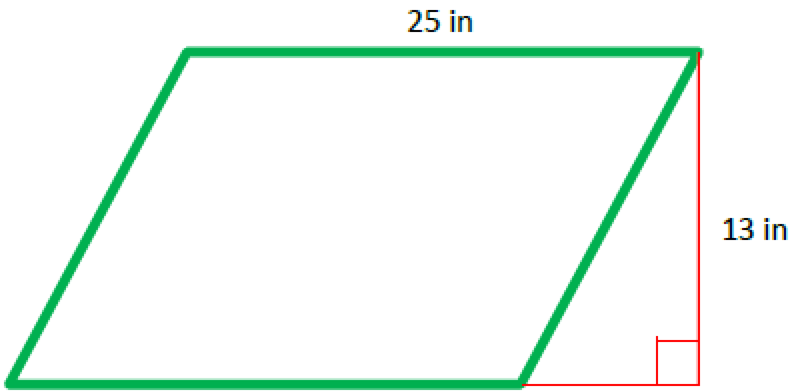
Area of Parallelogram ABCD =

$24\ cm^2 + 24\ cm^2 = 48\ cm^2$

Since the two triangles are congruent, so as their areas. The area of a parallelogram can also be obtained by multiplying the area of one of the triangles by 2.

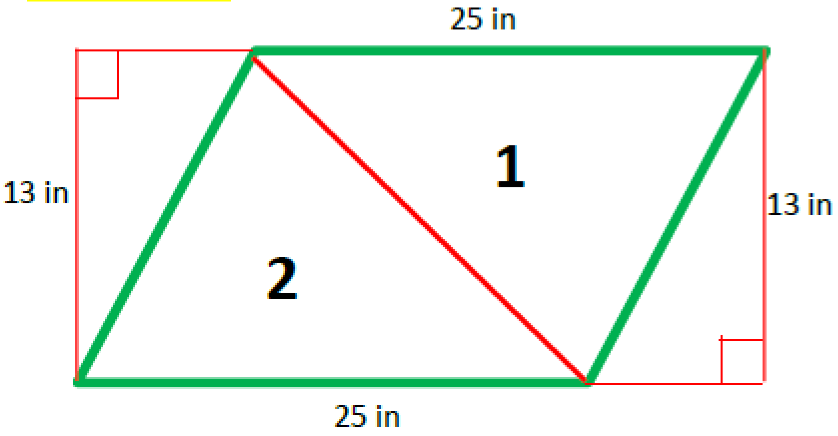
Sample Problem 4:

Find the area of the parallelogram below using triangles.



The Area of Polygons Through Composition and Decomposition

Solution:



Area of Triangle 1

$$A = \frac{1}{2}bh \text{ or } A = \frac{bh}{2}$$

$$A = \frac{(25)(13)}{2} = \frac{325}{2} = 162.5 \text{ in}^2$$

Area of Triangle 2

$$A = \frac{1}{2}bh \text{ or } A = \frac{bh}{2}$$

$$A = \frac{(25)(13)}{2} = \frac{325}{2} = 162.5 \text{ in}^2$$

Therefore,

Area of the parallelogram =

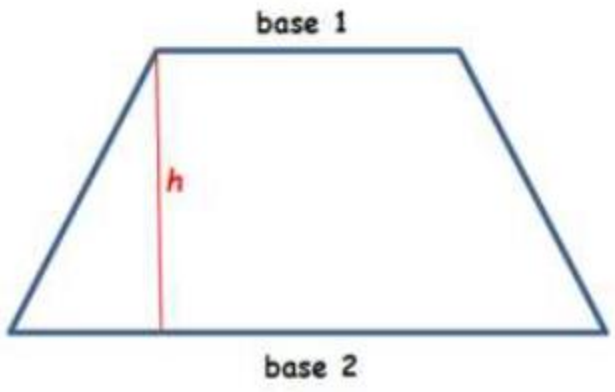
Area of Triangle 1 + Area of Triangle 2

Area of the parallelogram =

$162.5\ in^2 + 162.5\ in^2 = \mathbf{325\ in^2}$

Decomposing Trapezoids

The area of a trapezoid can be determined by finding the average of the two bases and multiplying the answer by the given height.



The area of a trapezoid:

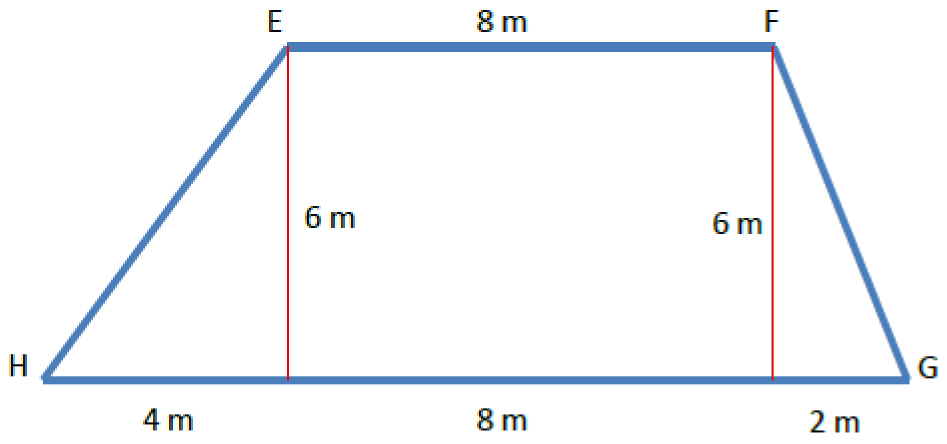
$$A = \frac{(b_1+b_2)}{2} \times h \text{ or } A = \frac{(b_1+b_2)h}{2}$$

Decomposing Trapezoids

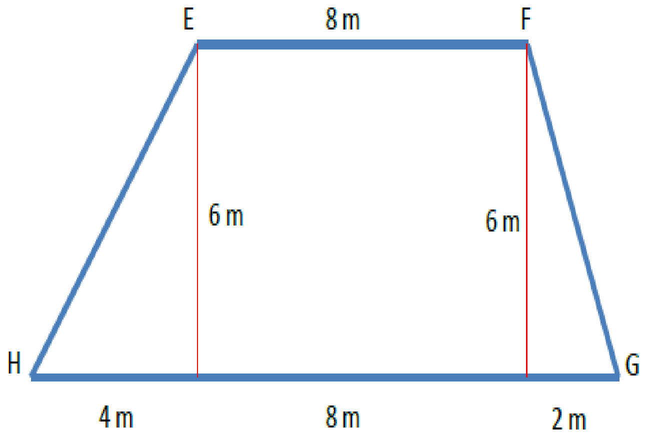
Aside from using the given formula, the area of trapezoids can be determined by composition and decomposition.

Example:

Below is Trapezoid EFGH with bases of lengths 8 m and 14 m respectively, with a height of 6m. Find its area.



The Area of Polygons Through Composition and Decomposition



Area of Triangle 1

$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

$A = \frac{(4)(6)}{2} = \frac{24}{2} = 12\ m^2$

Area of Triangle 2

$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

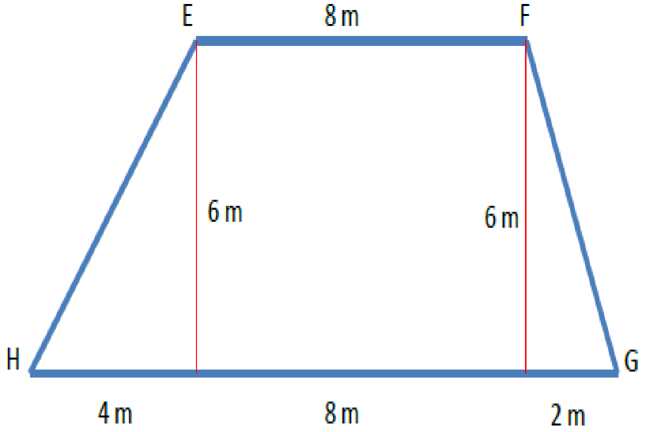
$A = \frac{(2)(6)}{2} = \frac{12}{2} = 6\ m^2$

Area of the Rectangle

$A = lw$

$A = 8 \times 6 = 48\ m^2$

The Area of Polygons Through Composition and Decomposition

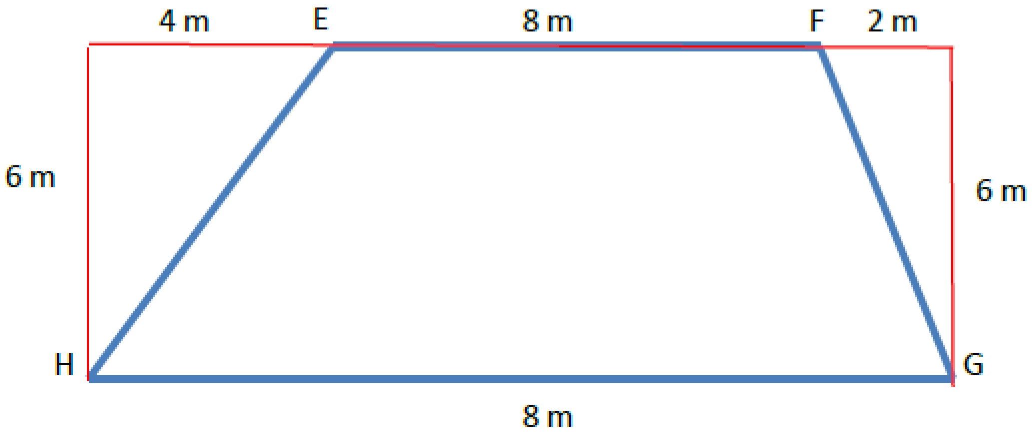


Area of the trapezoid = Area of Triangle 1 + Area of Triangle 2 + Area of the Rectangle

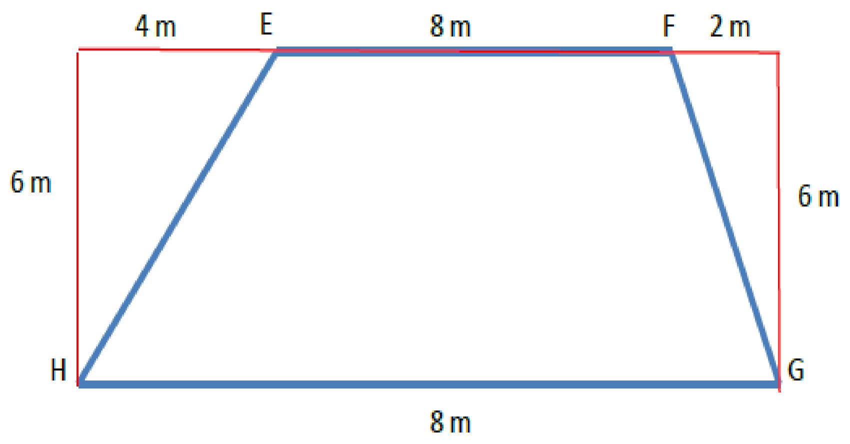
Area of the trapezoid = 12 m^2 + 6 m^2 + 48 m^2 = 66 m^2

But wait, there is another option:

If we surround the trapezoid with a rectangle, we can get its area by subtracting the area of the two triangles from the area of the rectangle.



The Area of Polygons Through Composition and Decomposition



Area of the Rectangle

$$A = lw$$

$$A = 14 \times 6 = 84 \text{ m}^2$$

Area of Triangle 1

$$A = \frac{1}{2}bh$$

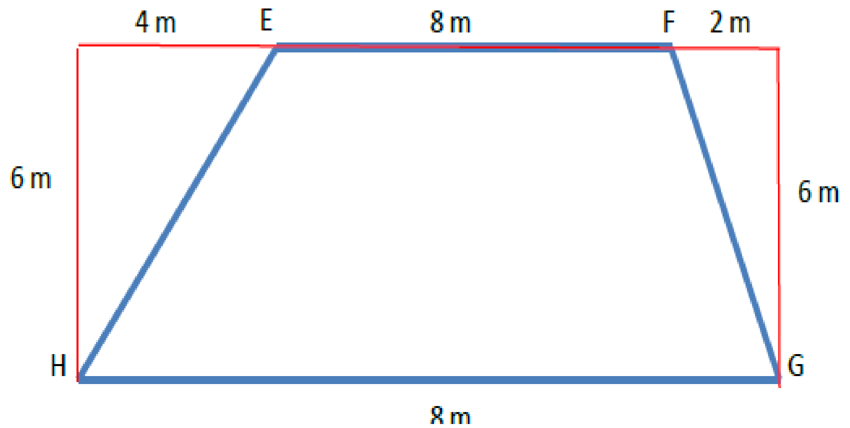
$$A = \frac{(4)(6)}{2} = \frac{24}{2} = 12 \text{ m}^2$$

Area of Triangle 2

$$A = \frac{1}{2}bh$$

$$A = \frac{(2)(6)}{2} = \frac{12}{2} = 6 \text{ m}^2$$

The Area of Polygons Through Composition and Decomposition



Area of the trapezoid = *Area of the Rectangle* – (*Area of Triangle 1* + *Area of Triangle 2*)

Area of the trapezoid = **84 m^2 – (12 m^2 + 6 m^2) = 66 m^2**

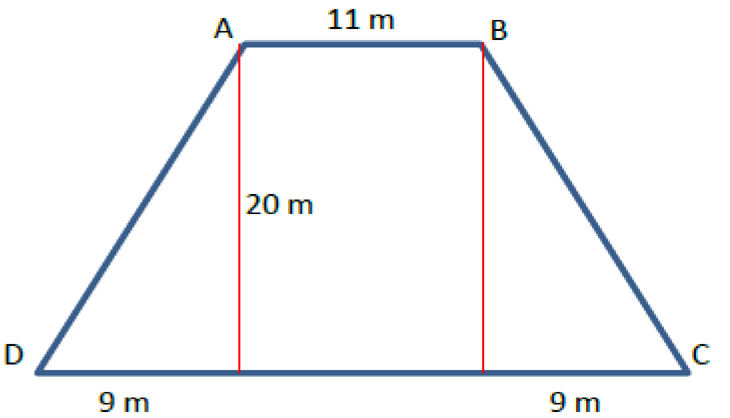
Notice that either way gives the same answer.

Sample Problem 5:

Given the Trapezoid ABCD, find the following:

a. CD = ?

b. Area of Trapezoid ABCD



Solution:

a. CD = 29 m

b. Area of the trapezoid = Area of Triangle 1 + Area of Triangle 2 + Area of the Rectangle

Area of Triangle 1	Area of Triangle 1	Area of the Rectangle
$A = \frac{1}{2}bh$	$A = \frac{1}{2}bh$	$A = lw$
$A = \frac{(9)(20)}{2} = \frac{180}{2} = 90m^2$	$A = \frac{(9)(20)}{2} = \frac{180}{2} = 90m^2$	$A = 20 \times 11 = 220m^2$

Area of the trapezoid = 90 m^2 + 90 m^2 + 220 m^2

Area of the trapezoid = 400 m^2