Classifying 2-Dimensional Shapes

This lesson covers the definition and the properties of triangles and trapezoids, as well as the classification of the same according to their sides and angles. Though trapezoids are classified under quadrilaterals, this lesson will only touch its different types. An in-depth lesson on quadrilaterals will be discussed in the next lesson.

Triangles

A triangle is a three-sided 2-dimensional shape that has the following parts:

- **three vertices**
- **three sides**
- **three angles**

**Vertices** - these are the corners or the points where the sides meet.

**Sides** - these are segments that make up the triangle.

**Angles** - these are formed by the intersection of the segments/sides of a triangle.
Sample Problem 1: Refer to the given triangle to complete the table.

<table>
<thead>
<tr>
<th>Vertices</th>
<th>Sides</th>
<th>Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The Sum of the Angles of a Triangle

The unit of measure of angles in any 2-dimensional shape is in **degrees**. In the case of triangles, the sum of the measures of the three angles is 180 degrees.

\[ m\angle A + m\angle B + m\angle C = 180^\circ \]
Sample Problem 2: Find the measure of $\angle E$. 

In the diagram, the triangle $DEF$ is given with angles $D$ and $F$ labeled as $64^\circ$ and $57^\circ$ respectively. The angle $E$ is to be found.
Classification of Triangles
Not all triangles are the same; they can be classified according to their sides and angles.

Classification of Triangles According to Sides
A triangle differs from another triangle in the measure of their sides. These triangles are classified as:

- **EQUILATERAL**
- **ISOSCELES**
- **SCALENE**

Equilateral Triangle
If all three sides of a triangle have the same measure, then the triangle is called an equilateral triangle.

\[ \triangle ABC \] on the right is an equilateral triangle since all its three sides have the same measure.
Isosceles Triangle
An isosceles triangle is a triangle with two sides (at least) that have the same measure.

△DEF on the right is an isosceles triangle since the measure of at least two of its sides are the same.

Scalene Triangle
A scalene triangle is a triangle that has three unequal sides.

△GHI on the right is a scalene triangle, since all its three sides don't have the same measure.
Sample Problem 3: Classify the following triangles by their sides.

1. 
2. 
3. 
4. 
5. 
6.
Classification of Triangles According to Angles

A triangle also differs from another triangle in the measure of their angles. These triangles are classified as:

- **ACUTE**
- **OBTUSE**
- **RIGHT**
- **EQUIANGULAR**

**ACUTE TRIANGLE**

An acute triangle is a triangle in which all three angles are acute. An acute angle is an angle the measures less than 90 degrees. $45^\circ, 63^\circ$ and $89^\circ$ are some examples of acute angles.

In $\triangle JKL$; $\angle J$, $\angle K$ and $\angle L$ measure less than 90 degrees. All three angles are acute. Therefore, $\triangle JKL$ is an acute triangle.
OBTUSE TRIANGLE
An obtuse triangle is a triangle whose one angle is obtuse. An obtuse angle is an angle the measures greater than 90 degrees but less than 180 degrees. $100^\circ, 155^\circ$ and $179^\circ$ are some examples of obtuse angles.

In $\triangle PQR$; $\angle Q$ is an obtuse angle because it measures greater than 90 degrees but less than 180 degrees, while $\angle P$ and $\angle R$ are acute. Therefore, $\triangle PQR$ is an obtuse triangle.

RIGHT TRIANGLE
A right triangle is a triangle whose one angle is a right angle. A right angle measures 90 degrees. The other two angles in a right triangle are acute.

In $\triangle MNO$; $\angle N$ is a right angle because it measures greater than 90 degrees. $\angle M$ and $\angle O$ are acute angles. Therefore, $\triangle MNO$ is a right triangle.
**EQUIANGULAR TRIANGLE**

An equiangular triangle has three equal angles and each angle measures 60 degrees.

In $\triangle ABC$; $\angle A$, $\angle B$ and $\angle C$ have the same measure. Therefore, $\triangle ABC$ is an equiangular triangle.
Sample Problem 4: Classify the following triangles by their angles.

1. 
2. 
3. 
4. 
5. 
6.
Important Facts About Triangles

Take note of the following important fact about the relationship of the angles and sides of a triangle.

1. An **equilateral triangle** has three equal sides. It also follows that all its three angles are equal. The measure of the sides may vary, but the measure of the angles of the triangle remains the same, 60 degrees. Therefore an **equilateral triangle** is also an **equiangular triangle**.

![Equilateral Triangle Diagram]
2. An **equilateral triangle** is also an **isosceles triangle**. The condition for a triangle to be isosceles is to have “at least” two sides that are equal. An **equilateral triangle** has three equal sides, so it satisfies the condition that qualifies it to be an **isosceles triangle**. So, an **equilateral triangle** is **always** an isosceles triangle, but an isosceles triangle can **sometimes** be equilateral.

3. A **right triangle** can **sometimes** be an **isosceles triangle** or a **scalene triangle**, vice versa.
4. An obtuse triangle can be an isosceles or a scalene triangle.

Sample Problem 5: Tell whether you agree to the given statement below. Explain your thoughts.

"An isosceles triangle is ALWAYS equilateral."

Justify your thoughts: ________________________________
______________________
______________________
______________________
Trapezoid

A trapezoid is a quadrilateral. A quadrilateral has four sides. It goes to say that trapezoids have 4 sides. But there is more to that. Below are the parts of trapezoids.

- four vertices
- four sides
- two upper base angles
- two lower base angles
- one pair of opposite sides that are parallel

<table>
<thead>
<tr>
<th>Vertices</th>
<th>Point A</th>
<th>Point B</th>
<th>Point C</th>
<th>Point D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Legs</td>
<td></td>
<td></td>
<td></td>
<td>$\overline{AD}$ and $\overline{BC}$</td>
</tr>
<tr>
<td>Upper Base Angles</td>
<td></td>
<td></td>
<td></td>
<td>$\angle A$ and $\angle B$</td>
</tr>
<tr>
<td>Lower Base Angles</td>
<td></td>
<td></td>
<td></td>
<td>$\angle C$ and $\angle D$</td>
</tr>
<tr>
<td>Pair of parallel sides</td>
<td></td>
<td></td>
<td></td>
<td>$\overline{AB}$ and $\overline{CD}$</td>
</tr>
</tbody>
</table>
The Sum of the Angles of a Trapezoid
The sum of the measures of the four angles of a trapezoid is 360 degrees.

Below is the relationship of the sum if the angles of a trapezoid:

- \( m\angle A + m\angle B + m\angle C + m\angle D = 360 \)
- \( m\angle A + m\angle D = 180 \)
- \( m\angle B + m\angle C = 180 \)

Sample Problem 6: Complete the table below.

<table>
<thead>
<tr>
<th>Vertices</th>
<th>Legs</th>
<th>Upper Base</th>
<th>Lower Base</th>
<th>Pair of parallel sides</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Classification of Trapezoids

Trapezoids are classified as follows:

- **RIGHT**
- **ISOSCELES**
- **SCALENE**

**RIGHT TRAPEZOID**

A right trapezoid has a pair of right angles.

In trapezoid MNOP, adjacent angles ∠M and ∠P are right angles, therefore it is a right trapezoid.

**ISOSCELES TRAPEZOID**

An isosceles trapezoid has equal legs. The lower base angles are equal and the upper base angles are equal.

In trapezoid VWXY, the legs WY and WX are equal. The measures of the upper base angles ∠V and ∠W are equal, and so as the lower base angles ∠Y and ∠X. Therefore it is an isosceles trapezoid.
**SCALENE TRAPEZOID**

A scalene trapezoid has no equal sides.

In trapezoid ABCD, none of the sides are equal, therefore the trapezoid is scalene.

Take note that a right trapezoid can be scalene, and vice versa.
Sample Problem 7: Classify the following trapezoids.

1. 

2. 

3. 

---

Copyright © MathTeacherCoach.com