

Unit 4 Lesson 6

Math 6

#### Students will be able to:

- Determine if the given expressions are equivalent given the value of the variable.
- Define and identify like terms.
- Generate equivalent expressions by combining like terms.
- State the distributive property.
- Write equivalent expressions in factored form using the greatest common factor and the distributive property.
- Use the distributive property to write equivalent expressions in standard form.



#### **Key Vocabulary:**

Like Terms

**Greatest Common Factor (GCF)** 

**Distributive Property** 

**Factored Form** 

**Expanded Form** 

Standard Form

**Equivalent Expressions** 



Equivalent Expressions are expressions that have the same value. They may look different but will have the same result if calculated. For example,  $\frac{5^2+2}{2}$  and  $\frac{9\times3}{2}$  are equivalent expressions. See why below:

$$5^2 + 2 = 25 + 2$$
  
= 27  
 $9 \times 3 = 27$ 

$$5^2 + 2 = 25 + 2$$
  
= 27  
 $9 \times 3 = 27$ 

The two expressions have the same answer, 27. Therefore, we can say that they are equivalent expressions.

$$5^2 + 2 = 9 \times 3$$

The same thing goes for expressions involving variables. Are  $\frac{5x+7}{4}$  and  $\frac{2x+3x+9-2}{4}$  equivalent expressions? To find out if they are, we can replace the variable  $\frac{x}{4}$  by number  $\frac{3}{4}$  and see if both expressions have the same result.

$$5x + 7 = (5)(3) + 7$$

$$= 15 + 7$$

$$= 22$$

$$2x + 3x + 9 - 2 = (2)(3) + (3(3) + 9 - 2)$$

$$= 6 + 9 + 9 - 2$$

$$= 22$$

## Equivalent Expressions

Remember that when a number is written next to a variable,

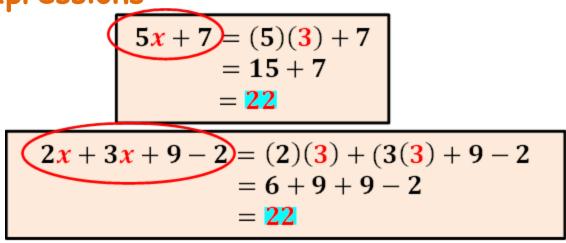
multiplication	•	in between, the o	peration is ALWAYS
	3 <i>a</i>	is the same as	$3 \cdot \alpha$

3 <i>a</i>	is the same as	$3 \cdot a$

5 <i>c</i>	is the same as	5 · <i>c</i>

<b>2</b> <i>m</i>	is the same as	$2 \cdot m$

Equivalent Expressions



It is very clear that the expressions  $\frac{5x}{4} + \frac{7}{7}$  and  $\frac{2x}{4} + \frac{3x}{4} + \frac{9}{4} + \frac{2}{3}$  are equivalent expressions because both have the same result,  $\frac{22}{4}$ . If the variable  $\frac{x}{4}$  is replaced by any number, the two expressions will remain equivalent.

Sample Problem 1: Determine if the given expressions are equivalent given the value of the variable.

a. 
$$3x-12$$
 and  $3(x-4)$ , for  $x=2$ 

b. 
$$14a - 6a \ \ and \ 8a, \ for \ a = 3$$



Sample Problem 1: Determine if the given expressions are equivalent given the value of the variable.

a. 
$$3x-12$$
 and  $3(x-4)$ , for  $x=2$ 

$$3x - 12 = 3 \times (2) - 12$$
  
= 6 - 12  
= -6  
$$3(x - 4) = 3(2 - 4)$$
  
= 3 × (-2) =

The expressions are equivalent.

b. 
$$14a - 6a \ \ and \ 8a, \ for \ a = 3$$

$$14a - 6a = 14 \times 3 - 6 \times 3$$
  
=  $42 - 18$   
=  $24$ 

The expressions are equivalent.

## Generating Equivalent Equations

To generate equivalent expressions you can use:

- 1. Combining like terms
- 2. Factoring
- 3. Distributive Property of Multiplication



#### Like Terms

Like terms are terms that have the same variables raised to the same power or exponent, and can have different coefficients. To simplify expressions, only like terms can be combined using the operations addition and subtraction.

Study the examples on the next slide:

#### Like Terms

Sample	Problem	2:	Circle	all	like	terms	in each	set.

5<sub>mp</sub>

**Equivalent Expressions** 

-10

-11mnp

2x

9m

-24xy

5.  $21x^2y^3z^2 - 3x^2y^3z^2 - 36x^3y^3z^2 + 42x^2y^2z^2 - x^2y^3z^2 + 39x^2y^3z^3$ 

 $-8x^3$ 

mnp

12xy

12

16mnp

-32yz

35ab<sup>3</sup>c

25m<sup>2</sup>np

100xy

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- 9xy  $25ab^2c$ 
  - -24xz  $14a^2bc$   $-28abc^2$   $-5ab^2c$

Sample Problem 2: Circle all like terms in each set.

- $-8x^3$ 2x
- (16mnp) (11mnp) 25m<sup>2</sup>np 5mp 9m mnp
- (-24xy) (12xy) -32yz -24xz
- 14a<sup>2</sup>bc -28abc<sup>2</sup>  $(-5ab^2c)$ 35ab<sup>3</sup>c
- $(-3x^2y^3z^2)$   $36x^3y^3z^2$   $42x^2y^2z^2$   $(-x^2y^3z^2)$

You can add up terms together to make a single term Study the examples and follow the steps as to how it is done.

1. Combine like terms in the expression  $\frac{4x}{4} + \frac{5}{5} - \frac{x}{4} + \frac{3}{4}$  to generate its equivalent expression.

1. Combine like terms in the expression  $\frac{4x}{+5} = \frac{x}{x} + \frac{3}{2}$  to generate its equivalent expression.

Step 1: Identify all like terms. You may organize them in a way that all like terms are identified. Take note to use the + and - just before the coefficient. A highlighter can come in handy too, or you can group all like terms together before combining them.

$$4x + 5 - x + 3$$
 or  $4x - x + 5 + 3$ 

Step 2: Combine the coefficients of like terms and then copy the variable. 4x - x + 5 + 3

The coefficient of 4x is 4 while the coefficient of -x is -1. To understand this better, we can replace the variable x with the word "apple":

4 apples minus 1 apple is equal to 3 apples

In the same way that we combine the coefficients and just carry the variable.

$$(4-1)x+5+3$$
  
 $3x+8$ 

Therefore, 
$$\frac{4x+5-x+3}{}$$
 and  $\frac{3x+8}{}$  are equivalent expressions.

2. Combine like terms in the expression  $\frac{5y}{y} + \frac{3x}{x} - \frac{4y}{y} - \frac{8x}{x}$  to generate its equivalent expression.

$$5y + 3x - 4y - 8x$$

$$-8x + 3x + 5y - 4y$$

$$(-8 + 3)x + (5 - 4)y$$

$$-5x + y$$

It is understood that the coefficient of y is 1 and there is no need to write it. Therefore,  $\frac{5y}{y} + \frac{3x}{x} - \frac{4y}{y} - \frac{8x}{x}$  and  $\frac{5x}{y} + \frac{y}{y}$  are equivalent expressions.

1. 
$$7m - 3m$$

$$2.5b - 9 + 4b + 6$$

3. 
$$4p + 10q - 5p - 7q$$
 4.  $-9 + 5g - 3h + 15 + 7h - 2g$ 

1. 
$$7m - 3m$$

$$7m - 3m$$
$$(7 - 3)m$$
$$4m$$

$$7m - 3m$$
 and  $4m$  are equivalent expressions.

$$2.5b - 9 + 4b + 6$$

$$5b + 4b - 9 + 6$$
  
 $(5+4)b - 9 + 6$   
 $9b - 3$ 

$$5b - 9 + 4b + 6$$
 and  $9b - 3$  are equivalent expressions.

3. 
$$4p + 10q - 5p - 7q$$
  
 $4p - 5p + 10q - 7q$   
 $(4 - 5)p + (10 - 7)q$   
 $(-1)p + (3)q$   
 $-p + 3q$ 

$$4p + 10q - 5p - 7q$$
 and  $-p + 3q$  are equivalent expressions.

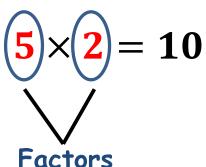
4. 
$$-9 + 5g - 3h + 15 + 7h - 2g$$
  
 $-9 + 5g - 3h + 15 + 7h - 2g$   
 $5g - 2g - 3h + 7h + 15 - 9$   
 $(5 - 2)g + (-3 + 7)h + (15 - 9)$   
 $3g + 4h + 6$ 

$$-9 + 5g - 3h + 15 + 7h - 2g$$
 and  $3g + 4h + 6$  are equivalent expressions.



## Factoring the GCF

Factoring is the process of getting the factors of any given product. Factors are the numbers or variable that you multiply and whatever the answer is the product.



The greatest common factor or GCF is the highest number that can divide two or more given numbers.

## Factoring the GCF

Or simply the greatest factor that is common in two or more given numbers. To get the GCF, we can either use the Listing Method or the Factor Tree.

Study the example below:

Find the GCF of 8 and 12.



#### Find the GCF of 8 and 12.

Listing Method - list all the factors of the given numbers. The greatest factor common to the given numbers is the GCF.

Factors of 8: 1, 2, **4**, 8

Factors of 12: 1, 2, 3, 4, 6, 12

The GCF of 8 and 12 is 4.

Factor Tree - is a tool used to break down any given number into its prime factors. Multiply the common prime factors to find the GCF.

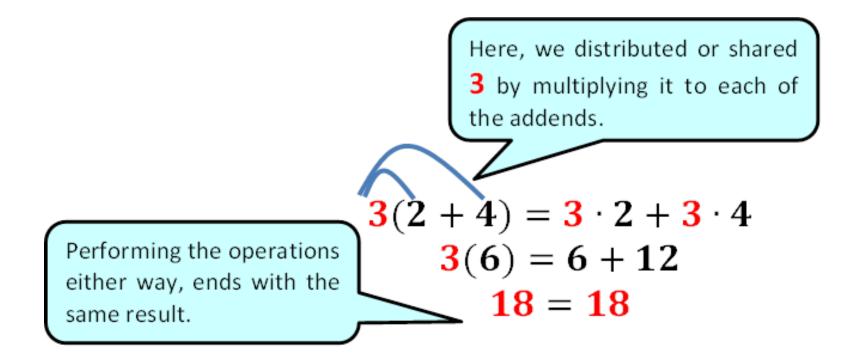
The GCF of 8 and 12 is 4.

### The Distributive Property of Multiplication

In Mathematics, the distributive property is used in rewriting expressions with equivalent expressions. Distributive comes from its root word "distribute" which means "to share".

This property states that multiplying a number by a sum or a difference is also the same as multiplying the number to the addends separately. Sounds confusing? Take a look at the example on the next sllide that will help us clear the air.

### The Distributive Property of Multiplication



### The Distributive Property of Multiplication

Replacing the numbers with variables, here is the distributive property of multiplication:

$$a(b+c)=ab+ac$$

By factoring the GCF and using the distributive property, we can generate equivalent expressions by rewriting them in factored form or expanded form. Study the examples below and take note of the steps.

1. 12m + 8n

**Step 1:** Expand each term of the expression using the prime factors of the coefficients.

$$3 \cdot 2 \cdot 2 \cdot m + 2 \cdot 2 \cdot 2 \cdot n$$

1.12m + 8n

**Step 2:** Determine the factors that are common in each term. This will be the GCF.

$$3 \cdot 2 \cdot 2 \cdot m + 2 \cdot 2 \cdot 2 \cdot n$$

The GCF in the expression is 4.

1. 12m + 8n

**Step 3:** The GCF is placed outside the parentheses and the rest will be placed inside the parentheses. Here, the distributive property is used.

$$\frac{2 \cdot 2(3 \cdot m + 2 \cdot n)}{4(3m + 2n)}$$

Therefore, 12m + 8n = 4(3m + 2n).

2. 
$$16a + 4$$

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot a + 2 \cdot 2$$
  
 $2 \cdot 2 \cdot 2 \cdot 2 \cdot a + 2 \cdot 2$ 

The GCF is 4.

$$2 \cdot 2(2 \cdot 2 \cdot a + 1)$$

2. 16a + 4

$$2 \cdot 2(2 \cdot 2 \cdot a + 1)$$

Why should there be a 1 inside the parentheses? Remember, multiplying a number by 1 is the number itself.

$$4(4a + 1)$$

Therefore, 16a + 4 = 4(4a + 1).

$$3.3pq - 5p$$

$$3 \cdot p \cdot q - 5 \cdot q$$

$$3 \cdot p \cdot q - 5 \cdot p$$

$$p(3 \cdot q - 5)$$

$$p(3q - 5)$$

Therefore, 
$$3pq - 5p = p(3q - 5)$$
.

# Generating Equivalent Expressions by Factoring the GCF and Using the Distributive Property

4.3mn - 3m

$$3 \cdot m \cdot n - 3 \cdot m$$
  
 $3 \cdot m \cdot n - 3 \cdot m$   
 $3 \cdot m(n-1)$ 

Therefore,  $3mn - 3m = 3 \cdot m(n-1)$ .

1. 
$$2g + 10h$$

3. 
$$24vw + 8w$$

4. 
$$9b - 27bc$$

1. 
$$2g + 10h$$

$$2 \cdot g + 2 \cdot 5 \cdot h$$
  
 $2 \cdot g + 2 \cdot 5 \cdot h$   
 $2(g + 5 \cdot h)$   
 $2(g + 5h)$ 

$$2g + 10h = 2(g + 5h)$$

2. 
$$7p - 9pq$$

$$7 \cdot p - 3 \cdot 3 \cdot p \cdot q$$

$$7 \cdot p - 3 \cdot 3 \cdot p \cdot q$$

$$p(7 - 3 \cdot 3 \cdot q)$$

$$p(7 - 9q)$$

$$7p - 9pq = p(7 - 9q)$$

3. 
$$24vw + 8w$$

$$3 \cdot 2 \cdot 2 \cdot 2 \cdot v \cdot w + 2 \cdot 2 \cdot 2 \cdot w$$

$$3 \cdot 2 \cdot 2 \cdot 2 \cdot v \cdot w + 2 \cdot 2 \cdot 2 \cdot w$$

$$2 \cdot 2 \cdot 2 \cdot w (3 \cdot v + 1)$$

$$8w(3 \cdot v + 1)$$

$$8w(3v + 1)$$

$$24vw + 8w = 8w(3v + 1)$$

4. 
$$9b - 27bc$$

$$3 \cdot 3 \cdot b - 3 \cdot 3 \cdot 3 \cdot b \cdot c$$

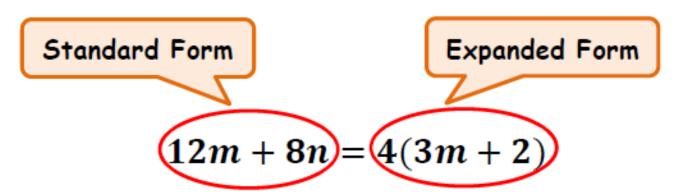
$$3 \cdot 3 \cdot b - 3 \cdot 3 \cdot 3 \cdot b \cdot c$$

$$3 \cdot 3 \cdot b (1 - 3 \cdot c)$$

$$9b(1 - 3c)$$

$$9b - 27bc = 9b(1 - 3c)$$

Using the distributive property we were able to find the expression equivalent to 12m+8n in factored or expanded form.



Now, we will do it the other way around. We will rewrite expressions in expanded form to its equivalent expression in standard form. Study the examples on the next slides and take note of the steps:

1. 
$$5(3x + 2y)$$

Remember that whatever is outside of the parenthesis is the greatest common factor of the expression. In this case, 5 is the GCF.

$$5(3x + 2y)$$

**Step 1:** Distribute the GCF by multiplying it to each term inside the parentheses.

$$(3x \cdot 5 + 2y \cdot 5)$$

Doing this removes the parentheses in the expression.

$$3x \cdot 5 + 2y \cdot 5$$

Step 2: To generate the standard form, find the product of each term.

$$15x + 10y$$

 $\frac{15x + 10y}{5}$  is now the standard form of  $\frac{5(3x + 2y)}{5}$ . These two are equivalent expressions.

2.3m(2n+1)

$$3m(2n + 1)$$
 $2n \cdot 3m + 1 \cdot 3m$ 
 $6mn + 3m$ 

Therefore, 
$$3m(2n+1) = 6mn + 3m$$
.

3.4ab(5c-4d)

$$4ab(5c-4d)$$
 $5c \cdot 4ab - 4d \cdot 4ab$ 
 $20abc - 16abd$ 

Therefore, 4ab(5c-4d) = 20abc-16abd.

1. 
$$7(m-2n)$$

2. 
$$a(3b + 8c)$$

3. 
$$5p(1+3q)$$
 4.  $8g(2h+5)$ 

1. 
$$7(m-2n)$$
 2.  $a(3b+8c)$ 

$$m \cdot 7 - 2n \cdot 7$$
 $7m - 14n$ 
 $3b \cdot a + 8c \cdot a$ 
 $3ab + 8ac$ 

$$7(m-2n)=7m-14n$$

$$a(3b + 8c) = 3ab + 8ac$$

3. 
$$5p(1+3q)$$

4. 
$$8g(2h+5)$$

$$1 \cdot \frac{5p}{5p} + 3q \cdot \frac{5p}{5p}$$
$$5p + 15pq$$

$$2h \cdot \textcolor{red}{\mathbf{8g}} + 5 \cdot \textcolor{red}{\mathbf{8g}} \\ \textcolor{blue}{\mathbf{16gh}} + \textcolor{blue}{\mathbf{40g}}$$

$$5p(1+3q) = 5p + 15pq$$

$$8g(2h+5) = 16gh + 40g$$