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Equivalent Expressions

Unit 4 Lesson 6

Math 6

Students will be able to:

- Determine if the given expressions are equivalent given the value of the variable.
- Define and identify like terms.
- Generate equivalent expressions by combining like terms.
- State the distributive property.
- Write equivalent expressions in factored form using the greatest common factor and the distributive property.
- Use the distributive property to write equivalent expressions in standard form.

Key Vocabulary:

Like Terms

Greatest Common Factor (GCF)

Distributive Property

Factored Form

Expanded Form

Standard Form

Equivalent Expressions

Equivalent Expressions

Equivalent Expressions are expressions that have the same value. They may look different but will have the same result if calculated. For example, $5^2 + 2$ and 9×3 are equivalent expressions. See why below:

$$5^2 + 2 = 25 + 2$$
$$= 27$$

$$9 \times 3 = 27$$

Equivalent Expressions

$$5^2 + 2 = 25 + 2$$
$$= 27$$

$$9 \times 3 = 27$$

The two expressions have the same answer, 27. Therefore, we can say that they are equivalent expressions.

$$5^2 + 2 = 9 \times 3$$

Equivalent Expressions

The same thing goes for expressions involving variables. Are $5x + 7$ and $2x + 3x + 9 - 2$ equivalent expressions? To find out if they are, we can replace the variable x by number 3 and see if both expressions have the same result.

$$\begin{aligned} 5x + 7 &= (5)(3) + 7 \\ &= 15 + 7 \\ &= 22 \end{aligned}$$

$$\begin{aligned} 2x + 3x + 9 - 2 &= (2)(3) + (3(3) + 9 - 2 \\ &= 6 + 9 + 9 - 2 \\ &= 22 \end{aligned}$$

Equivalent Expressions

Remember that when a number is written next to a variable, without any operation in between, the operation is ALWAYS multiplication.

$3a$	is the same as	$3 \cdot a$
$5c$	is the same as	$5 \cdot c$
$2m$	is the same as	$2 \cdot m$

Equivalent Expressions

$$5x + 7 = (5)(3) + 7$$
$$= 15 + 7$$
$$= 22$$

$$2x + 3x + 9 - 2 = (2)(3) + (3(3) + 9 - 2$$
$$= 6 + 9 + 9 - 2$$
$$= 22$$

It is very clear that the expressions $5x + 7$ and $2x + 3x + 9 - 2$ are equivalent expressions because both have the same result, 22. If the variable x is replaced by any number, the two expressions will remain equivalent.

Sample Problem 1: Determine if the given expressions are equivalent given the value of the variable.

a. $3x - 12 \quad and \; 3(x - 4)$, for $x = 2$

b. $14a - 6a \quad and \; 8a$, for $a = 3$

Sample Problem 1: Determine if the given expressions are equivalent given the value of the variable.

a. $3x - 12 \quad and \; 3(x - 4)$, for $x = 2$

$3x - 12 = 3 \times (2) - 12$
$= 6 - 12$
$= -6$

$3(x - 4) = 3(2 - 4)$
$= 3 \times (-2) =$
$= -6$

The expressions are equivalent.

b. $14a - 6a \;\; and \; 8a$, for $a = 3$

$14a - 6a = 14 \times 3 - 6 \times 3$
$= 42 - 18$
$= 24$

$8a = 8 \times 3$
$= 24$

The expressions are equivalent.

Generating Equivalent Equations

To generate equivalent expressions you can use:

1. Combining like terms
2. Factoring
3. Distributive Property of Multiplication

Like Terms

Like terms are terms that have **the same variables raised to the same power or exponent**, and can have different coefficients. To simplify expressions, only like terms can be combined using the operations addition and subtraction.

Study the examples on the next slide :

Like Terms

$3b$, $2b$, $-7b$, $10b$, $-45b$

pr, $-5pr$, $11\ pr$, $27pr$

$4x^2y$, $12\ x^2y$, $-8x^2y$

$27cde^3$, $-16cde^3$, $4cde^3$

→ Like Terms

Sample Problem 2: Circle all like terms in each set.

1.	4	-10	$2x$	$-8x^3$	12	1
2.	$5mp$	$-11mnp$	$9m$	mnp	$16mnp$	$25m^2np$
3.	$9xy$	$-24xz$	$-24xy$	$12xy$	$-32yz$	$100xy$
4.	$25ab^2c$	$14a^2bc$	$-28abc^2$	$-5ab^2c$	$35ab^3c$	$15ab^2c$
5.	$21x^2y^3z^2$	$-3x^2y^3z^2$	$36x^3y^3z^2$	$42x^2y^2z^2$	$-x^2y^3z^2$	$39x^2y^3z^3$

Sample Problem 2: Circle all like terms in each set.

1.	4	-10	$2x$	$-8x^3$	12	1
2.	$5mp$	$-11mnp$	$9m$	mnp	$16mnp$	$25m^2np$
3.	$9xy$	$-24xz$	$-24xy$	$12xy$	$-32yz$	$100xy$
4.	$25ab^2c$	$14a^2bc$	$-28abc^2$	$-5ab^2c$	$35ab^3c$	$15ab^2c$
5.	$21x^2y^3z^2$	$-3x^2y^3z^2$	$36x^3y^3z^2$	$42x^2y^2z^2$	$-x^2y^3z^2$	$39x^2y^3z^3$

Generating Equivalent Expressions by Combining Like Terms

You can add up terms together to make a single term Study the examples and follow the steps as to how it is done.

1. Combine like terms in the expression $4x + 5 - x + 3$ to generate its equivalent expression.

Generating Equivalent Expressions by Combining Like Terms

1. Combine like terms in the expression $4x + 5 - x + 3$ to generate its equivalent expression.

Step 1: Identify all like terms. You may organize them in a way that all like terms are identified. Take note to use the + and – just before the coefficient. A highlighter can come in handy too, or you can group all like terms together before combining them.

$$4x + 5 - x + 3 \text{ or } 4x - x + 5 + 3$$

Generating Equivalent Expressions by Combining Like Terms

Step 2: Combine the coefficients of like terms and then copy the variable. $4x - x + 5 + 3$

The coefficient of $4x$ is 4 while the coefficient of $-x$ is -1. To understand this better, we can replace the variable x with the word "apple":

4 apples minus **1 apple** is equal to **3 apples**

Generating Equivalent Expressions by Combining Like Terms

In the same way that we combine the coefficients and just carry the variable.

$$(4-1)x+5+3$$
$$3x+8$$

Therefore, $4x+5-x+3$ and $3x+8$

are **equivalent expressions**.

Generating Equivalent Expressions by Combining Like Terms

2. Combine like terms in the expression $5y + 3x - 4y - 8x$ to generate its equivalent expression.

$$5y + 3x - 4y - 8x$$
$$-8x + 3x + 5y - 4y$$
$$(-8 + 3)x + (5 - 4)y$$
$$-5x + y$$

It is understood that the coefficient of y is 1 and there is no need to write it. Therefore, $5y + 3x - 4y - 8x$ and $-5x + y$ are **equivalent expressions**.

Sample Problem 3: Combine like terms to generate equivalent expressions.

1. $7m - 3m$

2. $5b - 9 + 4b + 6$

Sample Problem 3: Combine like terms to generate equivalent expressions.

3. $4p + 10q - 5p - 7q$

4. $-9 + 5g - 3h + 15 + 7h - 2g$

Sample Problem 3: Combine like terms to generate equivalent expressions.

1. $7m - 3m$

$7m - 3m$
$(7 - 3)m$
$4m$

$7m - 3m$ and $4m$ are equivalent expressions.

2. $5b - 9 + 4b + 6$

$5b + 4b - 9 + 6$
$(5 + 4)b - 9 + 6$
$9b - 3$

$5b - 9 + 4b + 6$ and $9b - 3$ are equivalent expressions.

Sample Problem 3: Combine like terms to generate equivalent expressions.

3. $4p + 10q - 5p - 7q$

$4p - 5p + 10q - 7q$
$(4 - 5)p + (10 - 7)q$
$(-1)p + (3)q$
$-p + 3q$

$4p + 10q - 5p - 7q$ and $-p + 3q$ are equivalent expressions.

4. $-9 + 5g - 3h + 15 + 7h - 2g$

$-9 + 5g - 3h + 15 + 7h - 2g$
$5g - 2g - 3h + 7h + 15 - 9$
$(5 - 2)g + (-3 + 7)h + (15 - 9)$
$3g + 4h + 6$

$-9 + 5g - 3h + 15 + 7h - 2g$ and $3g + 4h + 6$ are equivalent expressions.

Factoring the GCF

Factoring is the process of getting the **factors** of any given **product**. Factors are the numbers or variable that you multiply and whatever the answer is the product.

$$5 \times 2 = 10$$

Factors

The **greatest common factor** or **GCF** is the highest number that can divide two or more given numbers.

Factoring the GCF

Or simply the greatest factor that is common in two or more given numbers. To get the GCF, we can either use the **Listing Method** or the **Factor Tree**.

Study the example below:

Find the GCF of 8 and 12.

Find the GCF of 8 and 12.

Listing Method - list all the factors of the given numbers. The greatest factor common to the given numbers is the GCF.

Factors of 8: 1, 2, **4**, 8

Factors of 12: 1, 2, 3, **4**, 6, 12

The GCF of 8 and 12 is 4.

Factor Tree - is a tool used to break down any given number into its prime factors. Multiply the common prime factors to find the GCF.

The GCF of 8 and 12 is 4.

The Distributive Property of Multiplication

In Mathematics, the **distributive property** is used in rewriting expressions with equivalent expressions. Distributive comes from its root word "**distribute**" which means "**to share**".

This property states that **multiplying a number by a sum or a difference is also the same as multiplying the number to the addends separately**. Sounds confusing? Take a look at the example on the next sllide that will help us clear the air.

The Distributive Property of Multiplication

Here, we distributed or shared 3 by multiplying it to each of the addends.

$$3(2+4) = 3 \cdot 2 + 3 \cdot 4$$

$$3(6) = 6 + 12$$

$$18 = 18$$

Performing the operations either way, ends with the same result.

The Distributive Property of Multiplication

Replacing the numbers with variables, here is **the distributive property of multiplication**:

$$a(b + c) = ab + ac$$

Generating Equivalent Expressions by Factoring the GCF and Using the Distributive Property

By **factoring the GCF** and **using the distributive property**, we can generate equivalent expressions by rewriting them in **factored form** or **expanded form**. Study the examples below and take note of the steps.

1. $12m + 8n$

Step 1: Expand each term of the expression using the prime factors of the coefficients.

$$3 \cdot 2 \cdot 2 \cdot m + 2 \cdot 2 \cdot 2 \cdot n$$

Generating Equivalent Expressions by Factoring the GCF and Using the Distributive Property

1. $12m + 8n$

Step 2: Determine the factors that are common in each term. This will be the GCF.

$$3 \cdot 2 \cdot 2 \cdot m + 2 \cdot 2 \cdot 2 \cdot n$$

The GCF in the expression is 4.

Generating Equivalent Expressions by Factoring the GCF and Using the Distributive Property

1. $12m + 8n$

Step 3: The GCF is placed outside the parentheses and the rest will be placed inside the parentheses. Here, the distributive property is used.

$$2 \cdot 2(3 \cdot m + 2 \cdot n)$$

$$4(3m + 2n)$$

Therefore, $12m + 8n = 4(3m + 2n)$.

Generating Equivalent Expressions by Factoring the GCF and Using the Distributive Property

2. $16a + 4$

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot a + 2 \cdot 2$$

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot a + 2 \cdot 2$$

The GCF is 4.

$$2 \cdot 2(2 \cdot 2 \cdot a + 1)$$

Generating Equivalent Expressions by Factoring the GCF and Using the Distributive Property

2. $16a + 4$

$$2 \cdot 2(2 \cdot 2 \cdot a + 1)$$

Why should there be a 1 inside the parentheses? Remember, multiplying a number by 1 is the number itself.

$$4(4a + 1)$$

Therefore, $16a + 4 = 4(4a + 1)$.

Generating Equivalent Expressions by Factoring the GCF and Using the Distributive Property

3. $3pq - 5p$

$$3 \cdot p \cdot q - 5 \cdot q$$

$$3 \cdot p \cdot q - 5 \cdot p$$

$$p(3 \cdot q - 5)$$

$$p(3q - 5)$$

Therefore, $3pq - 5p = p(3q - 5)$.

Generating Equivalent Expressions by Factoring the GCF and Using the Distributive Property

4. $3mn - 3m$

$$3 \cdot m \cdot n - 3 \cdot m$$

$$3 \cdot m \cdot n - 3 \cdot m$$

$$3 \cdot m(n - 1)$$

Therefore, $3mn - 3m = 3 \cdot m(n - 1)$.

Sample Problem 4: Write equivalent expressions in factored form using the greatest common factor and the distributive property.

1. $2g + 10h$

2. $7p - 9pq$

Sample Problem 4: Write equivalent expressions in factored form using the greatest common factor and the distributive property.

3. $24vw + 8w$

4. $9b - 27bc$

Sample Problem 4: Write equivalent expressions in factored form using the greatest common factor and the distributive property.

1. $2g + 10h$

$$2 \cdot g + 2 \cdot 5 \cdot h$$
$$2 \cdot g + 2 \cdot 5 \cdot h$$
$$2(g + 5 \cdot h)$$
$$2(g + 5h)$$

$$2g + 10h = 2(g + 5h)$$

2. $7p - 9pq$

$$7 \cdot p - 3 \cdot 3 \cdot p \cdot q$$
$$7 \cdot p - 3 \cdot 3 \cdot p \cdot q$$
$$p(7 - 3 \cdot 3 \cdot q)$$
$$p(7 - 9q)$$

$$7p - 9pq = p(7 - 9q)$$

Sample Problem 4: Write equivalent expressions in factored form using the greatest common factor and the distributive property.

3. $24vw + 8w$

$$3 \cdot 2 \cdot 2 \cdot 2 \cdot v \cdot w + 2 \cdot 2 \cdot 2 \cdot w$$
$$3 \cdot 2 \cdot 2 \cdot 2 \cdot v \cdot w + 2 \cdot 2 \cdot 2 \cdot w$$
$$2 \cdot 2 \cdot 2 \cdot w(3 \cdot v + 1)$$
$$8w(3 \cdot v + 1)$$
$$8w(3v + 1)$$

$$24vw + 8w = 8w(3v + 1)$$

4. $9b - 27bc$

$$3 \cdot 3 \cdot b - 3 \cdot 3 \cdot 3 \cdot b \cdot c$$
$$3 \cdot 3 \cdot b - 3 \cdot 3 \cdot 3 \cdot b \cdot c$$
$$3 \cdot 3 \cdot b(1 - 3 \cdot c)$$
$$9b(1 - 3 \cdot c)$$
$$9b(1 - 3c)$$

$$9b - 27bc = 9b(1 - 3c)$$

Writing Equivalent Expressions from Expanded Form to Standard Form Using the Distributive Property

Using the distributive property we were able to find the expression equivalent to $12m + 8n$ in factored or expanded form.

Standard Form

Expanded Form

$$12m + 8n = 4(3m + 2)$$

Writing Equivalent Expressions from Expanded Form to Standard Form Using the Distributive Property

Now, we will do it the other way around. We will rewrite expressions in expanded form to its equivalent expression in standard form. Study the examples on the next slides and take note of the steps:

Writing Equivalent Expressions from Expanded Form to Standard Form Using the Distributive Property

1. $\mathbf{5(3x+2y)}$

Remember that whatever is outside of the parenthesis is the greatest common factor of the expression. In this case, **5** is the GCF.

$$\mathbf{5(3x+2y)}$$

Writing Equivalent Expressions from Expanded Form to Standard Form Using the Distributive Property

Step 1: Distribute the GCF by multiplying it to each term inside the parentheses.

$$(3x \cdot 5 + 2y \cdot 5)$$

Doing this removes the parentheses in the expression.

$$3x \cdot 5 + 2y \cdot 5$$

Writing Equivalent Expressions from Expanded Form to Standard Form Using the Distributive Property

Step 2: To generate the standard form, find the product of each term.

$$\mathbf{15x + 10y}$$

$\mathbf{15x + 10y}$ is now the standard form of $\mathbf{5(3x + 2y)}$. These two are equivalent expressions.

Writing Equivalent Expressions from Expanded Form to Standard Form Using the Distributive Property

2. $3m(2n+1)$

$$3m(2n+1)$$
$$2n \cdot 3m + 1 \cdot 3m$$
$$6mn + 3m$$

Therefore, $3m(2n+1) = 6mn + 3m$.

Writing Equivalent Expressions from Expanded Form to Standard Form Using the Distributive Property

3. $\mathbf{4ab(5c-4d)}$

$$4ab(5c-4d)$$
$$5c \cdot 4ab - 4d \cdot 4ab$$
$$20abc - 16abd$$

Therefore, $4ab(5c-4d) = 20abc - 16abd$.

Sample Problem 5: Use the distributive property to write equivalent expressions in standard form.

1. $7(m-2n)$

2. $a(3b+8c)$

Sample Problem 5: Use the distributive property to write equivalent expressions in standard form.

3. $5p(1 + 3q)$

4. $8g(2h + 5)$

Sample Problem 5: Use the distributive property to write equivalent expressions in standard form.

1. $7(m - 2n)$

$$m \cdot 7 - 2n \cdot 7$$
$$7m - 14n$$

$$7(m - 2n) = 7m - 14n$$

2. $a(3b + 8c)$

$$3b \cdot a + 8c \cdot a$$
$$3ab + 8ac$$

$$a(3b + 8c) = 3ab + 8ac$$

Sample Problem 5: Use the distributive property to write equivalent expressions in standard form.

3. $5p(1+3q)$

$$1 \cdot 5p + 3q \cdot 5p$$
$$5p + 15pq$$

$$5p(1+3q) = 5p + 15pq$$

4. $8g(2h+5)$

$$2h \cdot 8g + 5 \cdot 8g$$
$$16gh + 40g$$

$$8g(2h+5) = 16gh + 40g$$