Multiplication of Fractions and Whole Number

Unit 5 Lesson 2

Math 5

Students will be able to:

- Use fraction of a set to interpret fractions as division.

- Use tape diagrams to multiply a whole number by a fraction.

- Relate a fraction of a set to interpret fraction multiplication through repeated addition.

- Solve problems involving multiplication of factions and whole number using visual models or equations.

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Multiplication of Fractions and Whole Numbers

Key Vocabulary:

Fractions

Whole Numbers

Multiplication

Tape Diagram

Arrays

Numerator

Denominator

Finding Fractions of a Set

A **fraction** is a division of the **numerator** by the **denominator**. A fraction can be expressed as division and vice versa.
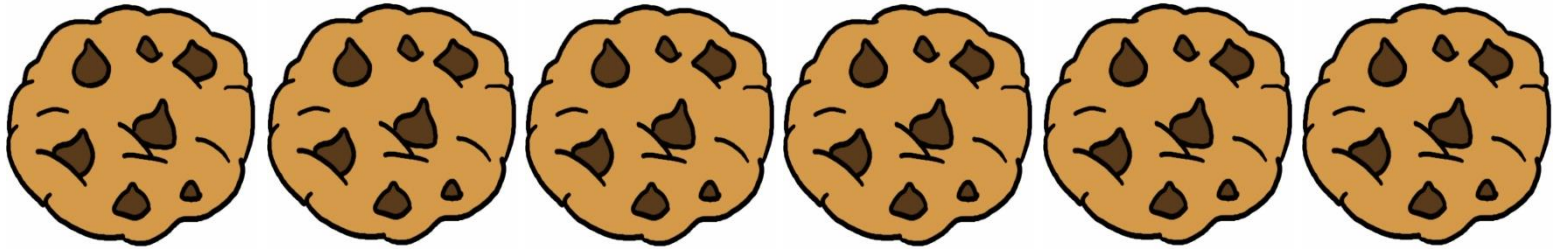
FRACTION	DIVISION
$$\dfrac{a}{b}$$ =	$$a \div b$$

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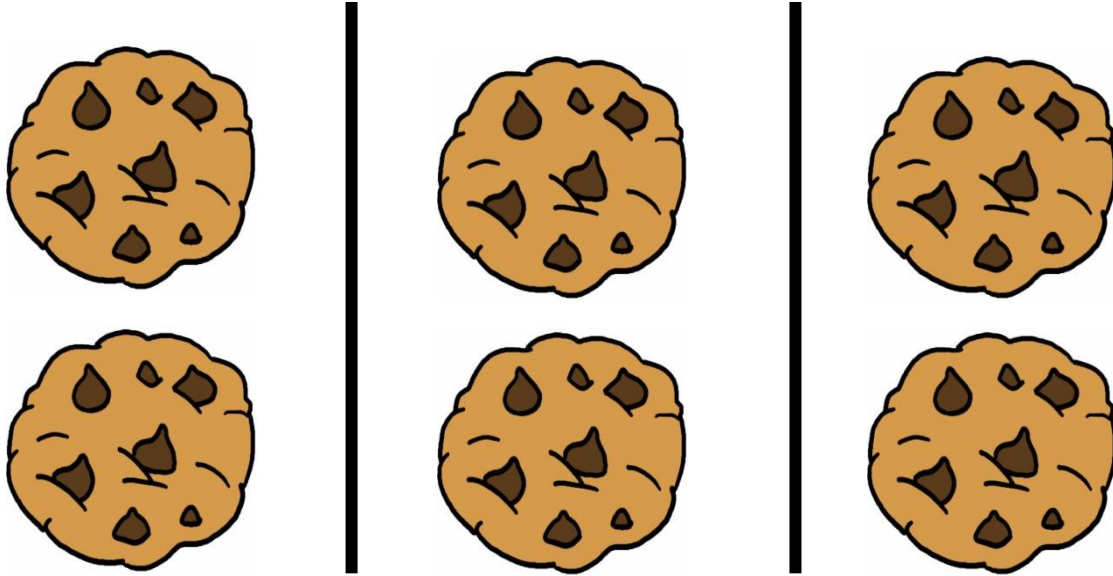# Finding Fractions of a Set

In this lesson, you will multiply fractions and whole numbers using arrays of figures or shapes to find fractions of a set.

If there are 6 cookies and you are to divide it into 3 parts equally, how many cookies will each part have?

Finding Fractions of a Set

Dividing the cookies, you'll get:

Finding Fractions of a Set

The division sentence for the given problem is:

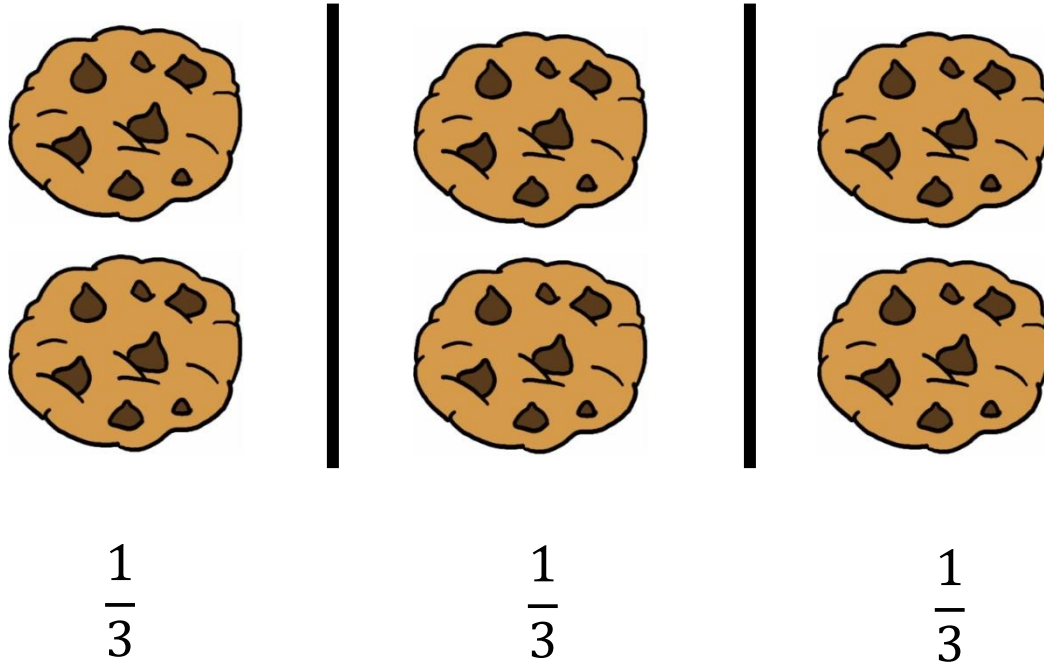$$6 \div 3 = 2$$

The division sentence expressed as a fraction is:

$$\frac{6}{3} = 2$$

If you want to know 1 third ($\frac{1}{3}$) of the cookies, how many cook will there be in each part?

Finding Fractions of a Set

Each part/group of cookies divided equally is one-third of the entire cookies.

$$\frac{1}{3}$$ $$\frac{1}{3}$$ $$\frac{1}{3}$$

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Finding Fractions of a Set

This shows that when you divide 6 by 3, there are 2 in each group, therefore:

$\frac{1}{3}$ **of 6** cookies is equal to **2 cookies**.

$\frac{2}{3}$ **of 6** cookies is equal to **4 cookies.**

$\frac{3}{3}$ **of 6** cookies is equal to **6 cookies.**

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Multiplication of Fractions and Whole Numbers

Example 1: What is $\frac{1}{3}$ of 12?

Make an array of 12 shapes (of the same kind) and divide it equally into three parts.
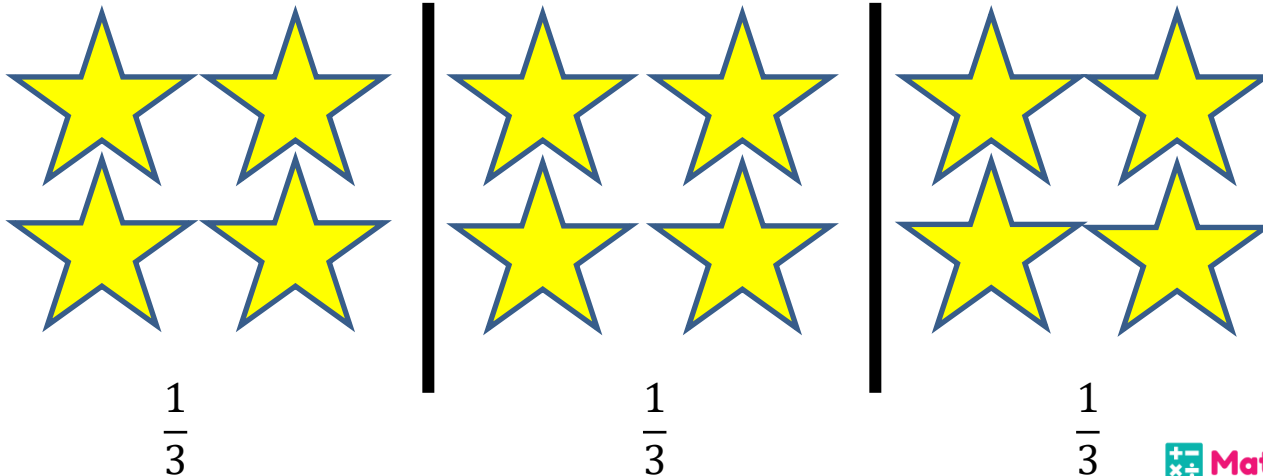
Multiplication of Fractions and Whole Numbers

The division sentence expressed as a fraction is:

$$\frac{12}{3} = 4$$

Each part/group of the stars divided equally is 1 third of the 24 stars.

$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$

Multiplication of Fractions and Whole Numbers

Therefore,

$\frac{1}{3}$ of 12 = 4

$\frac{2}{3}$ of 12 = 8

$\frac{3}{3}$ of 12 = 12

12 divided into 3 equal parts,

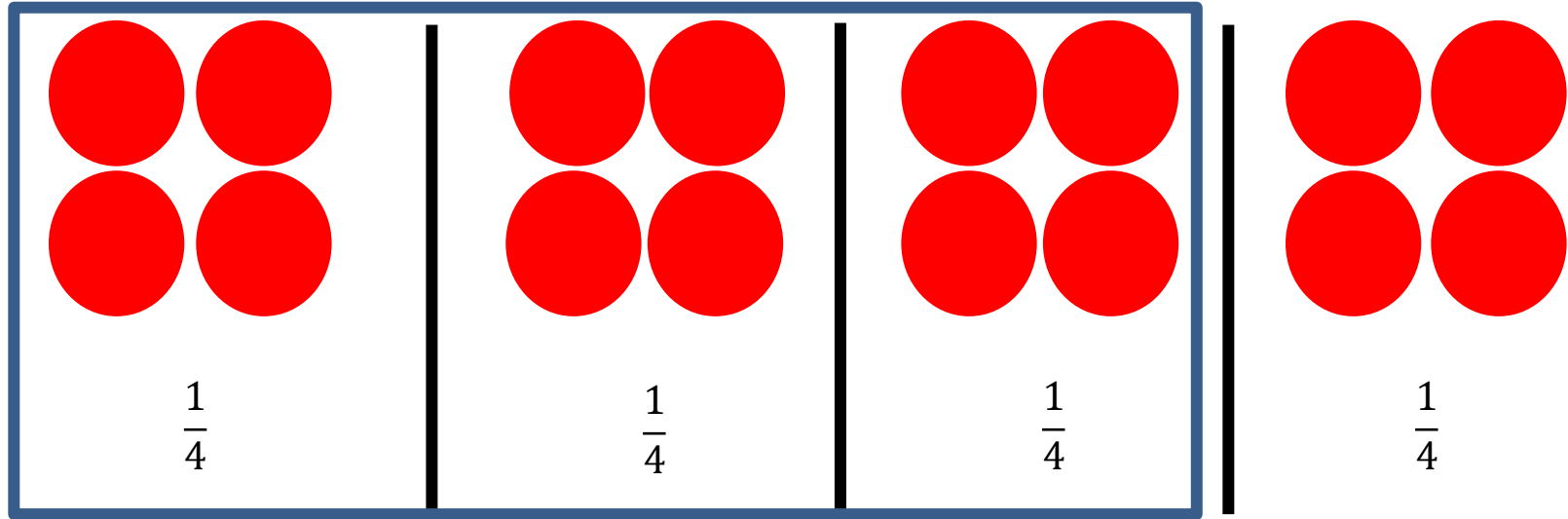each part or group is equal to 3.

$\frac{1}{3}$ of 12 = 4

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Example 2: What is $\frac{3}{4}$ of 16?

Draw 16 any shape of the same kind, and divide it equally into 6 groups.

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

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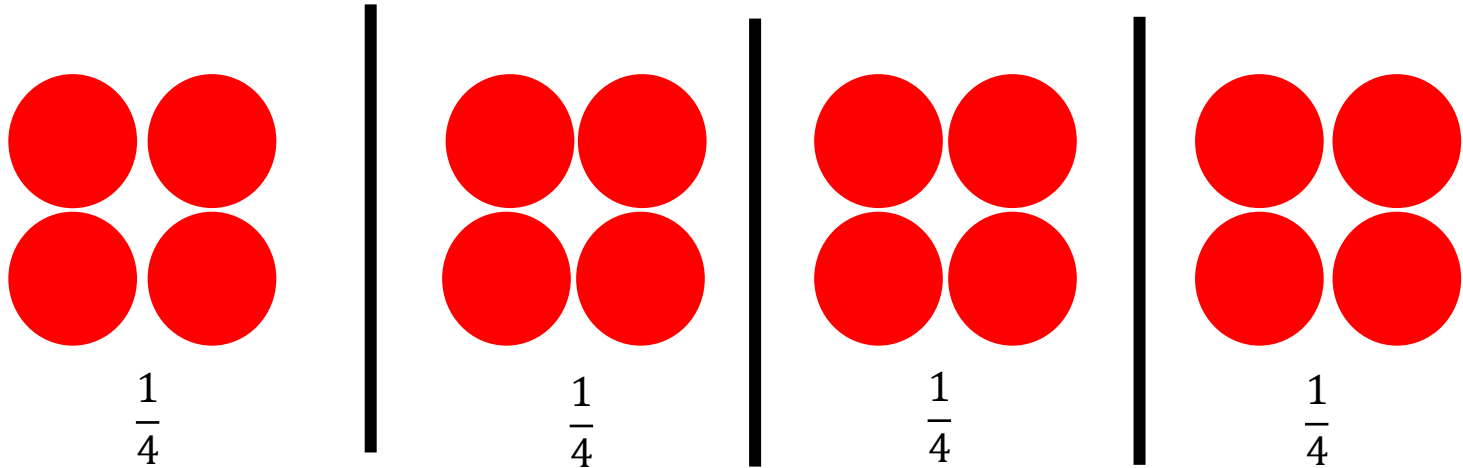
Multiplication of Fractions and Whole Numbers

Here is three-fourths of 16:

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

Therefore, $\frac{3}{4}$ of 16 = 12.

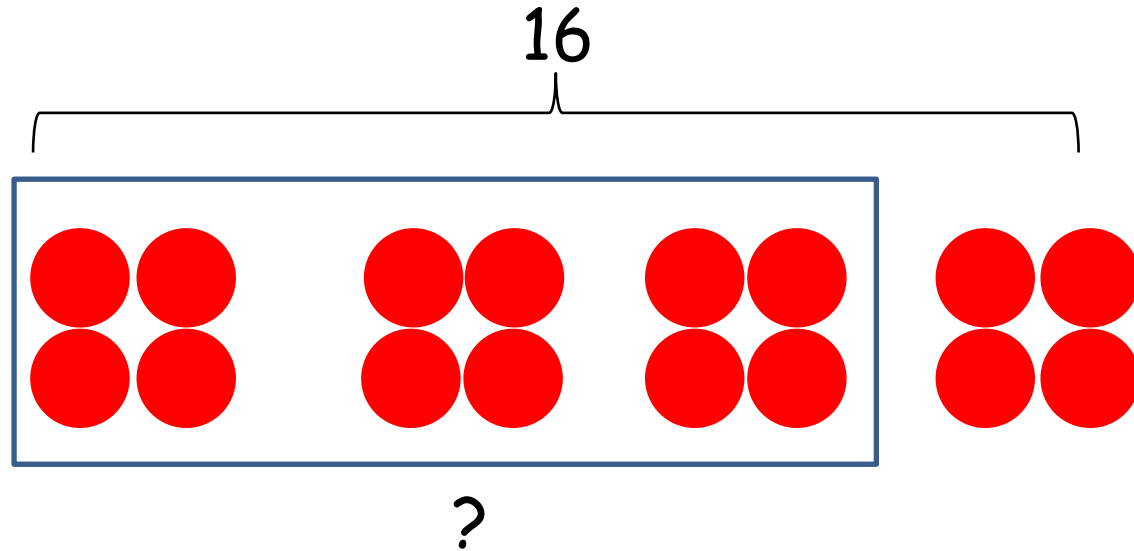MathTeacherCoach.com

Multiplication of Fractions and Whole Numbers

The problem can also be solved this way.

What is $\frac{3}{4}$ of 16?

16

?

Multiplication of Fractions and Whole Numbers
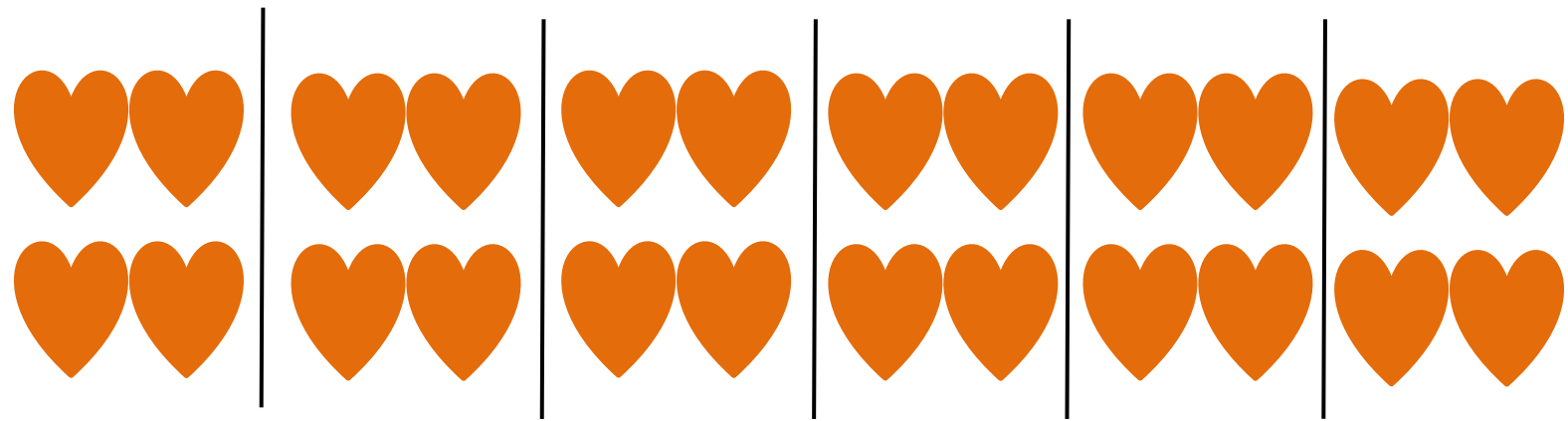
$$16 \div 4 = 4$$

$\frac{1}{4}$ of 16 = 4

If 16 is divided equally into 4 parts, each part is equal to 4. Since you are looking for 3 parts and you know that each part is 4, then:

3 x 4 = 12

$\frac{3}{4}$ of 16 = 12

Example 3: Find the value of the following.

Multiplication of Fractions and Whole Numbers

a. $\frac{1}{6}$ of 24 = ?

Divide 24 into 6 equal parts: $24 \div 6 = 4$

Each part is 1 sixth of 24.

Therefore, $\frac{1}{6}$ of 24 = 4

b. $\frac{3}{6}$ of 24 = ?

Each part is 1 sixth of 24.

$\frac{1}{6}$ of 24 = 4

Therefore, $\frac{3}{6}$ of 24 = 12.

c. $\frac{5}{6}$ of 24 = ?

Each part is 1 sixth of 24.

$\frac{1}{6}$ of 24 = 4

Therefore, $\frac{5}{6}$ of 24 = 20.

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Multiplication of Fractions and Whole Numbers

Sample Problem 1:

A. Draw an array of circles to find the value of the following.

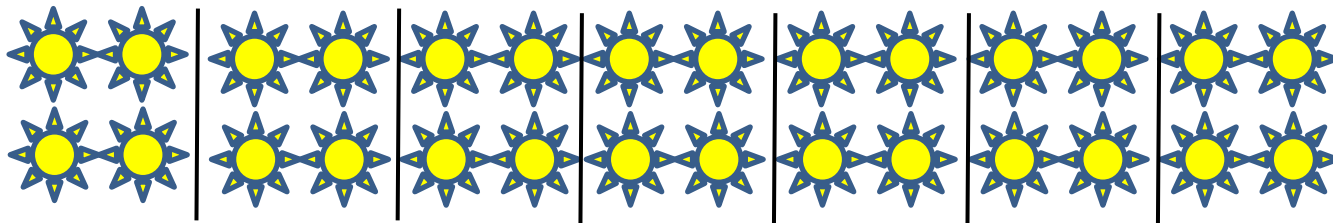1. $\frac{1}{5}$ of 35 = ?

2. $\frac{4}{6}$ of 36 = ?

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Multiplication of Fractions and Whole Numbers

Sample Problem 1:

B. Find the value of the following.

1. $\frac{1}{7}$ of 28 = ?

2. $\frac{4}{7}$ of 28 = ?

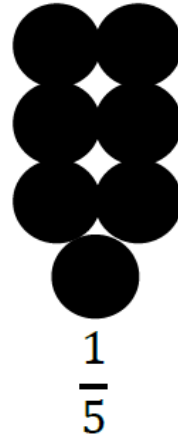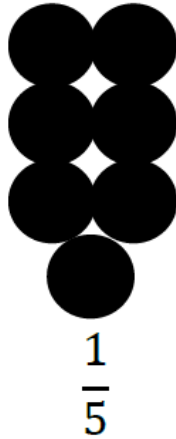3. $\frac{3}{7}$ of 28 = ?
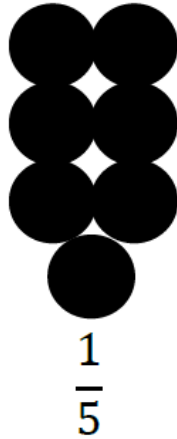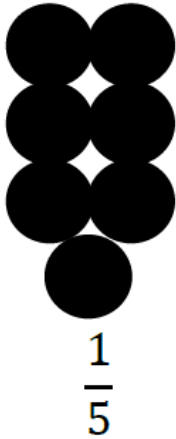
4. $\frac{6}{7}$ of 28 = ?

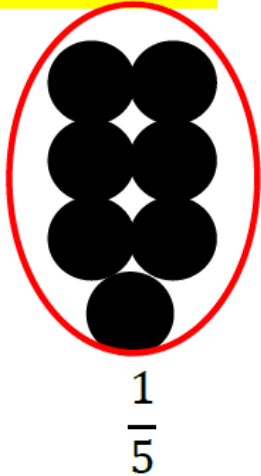5. $\frac{5}{7}$ of 28 = ?

Multiplication of Fractions and Whole Numbers

A. Draw an array of circles to find the value of the following.

1. $\frac{1}{5}$ of 35 = 7

$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$

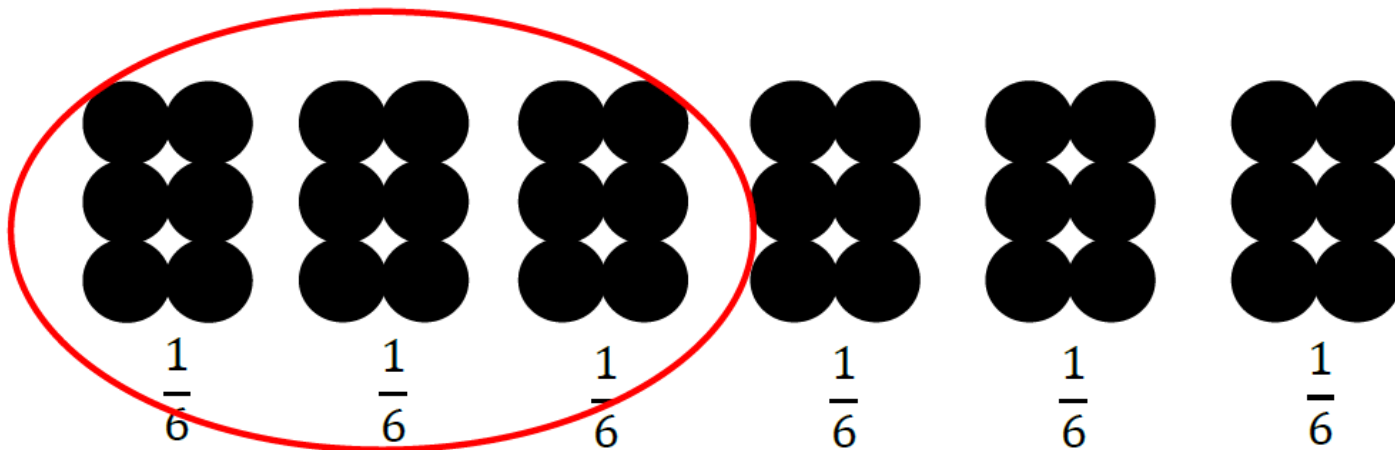MathTeacherCoach.com

Multiplication of Fractions and Whole Numbers

<mark>Solution:</mark>

A. Draw an array of circles to find the value of the following.

2. $\frac{4}{6}$ of 36 = 18

$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

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Multiplication of Fractions and Whole Numbers

B. Find the value of the following.
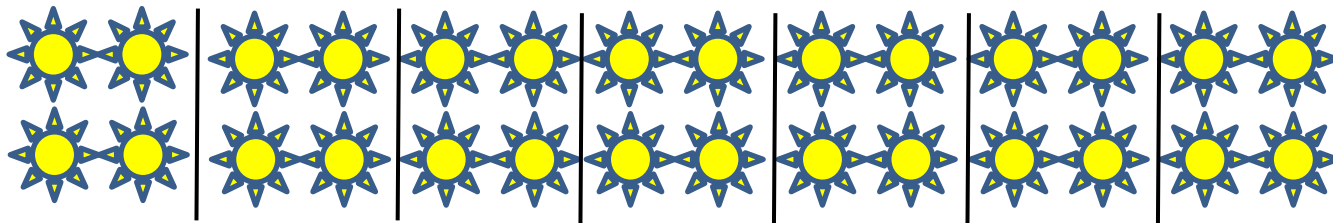
1. $\frac{1}{7}$ of 28 = 4
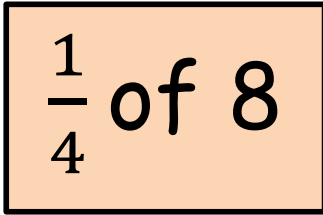
3. $\frac{3}{7}$ of 28 = 12

5. $\frac{5}{7}$ of 28 = 20

2. $\frac{4}{7}$ of 28 = 16

4. $\frac{6}{7}$ of 28 = 24
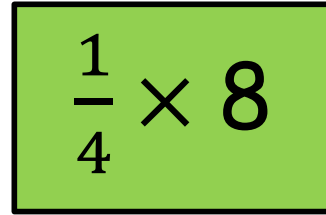
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Multiplying Fractions by Whole Numbers Using Tape Diagrams

Previously, you created arrays of shapes to show how to find a part of a whole, which is exactly the same as multiplying a fraction to a whole number, example:

$\frac{1}{4}$ of 8   is the same as $\frac{1}{4} \times 8$

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Multiplying Fractions by Whole Numbers Using Tape Diagrams

This time you will multiply fractions and whole numbers using tape diagrams. How is it done?

Examples:

a. Using a tape diagram, find $\frac{3}{7}$ of 42.

What is $\frac{3}{7}$ of 42?

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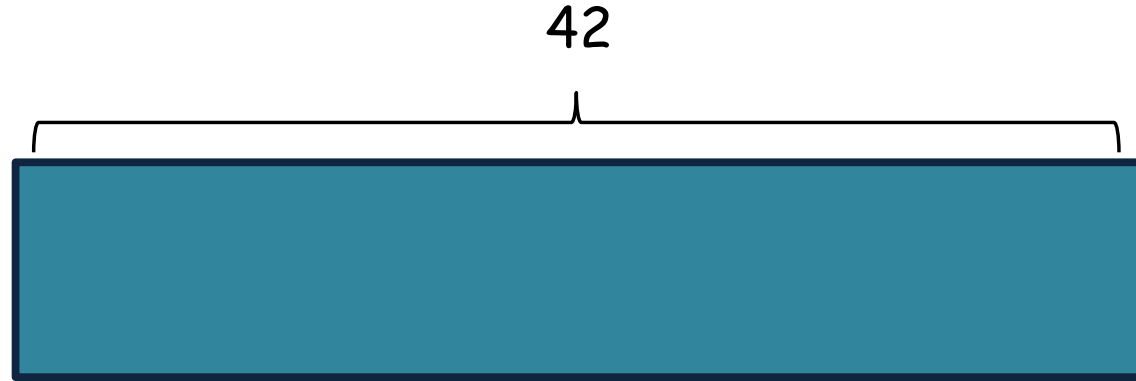# Multiplying Fractions by Whole Numbers Using Tape Diagrams

Step 1:   Here, you need to find 3 sevenths of 42. First, determine the whole in the given problem.

$$\frac{3}{7} \text{ of } \boxed{42} = ? \quad \boxed{42 \text{ is the whole!}}$$

Multiplying Fractions by Whole Numbers Using Tape Diagrams

Step 2:   Draw a bar that will represent the whole.

42

Multiplying Fractions by Whole Numbers Using Tape Diagrams

Step 3: Since we want $\frac{3}{7}$ of 42, the denominator in $\frac{3}{7}$ tells us how many units we need to cut the tape diagram. Thus, we need to cut the bar into 7 equal parts.
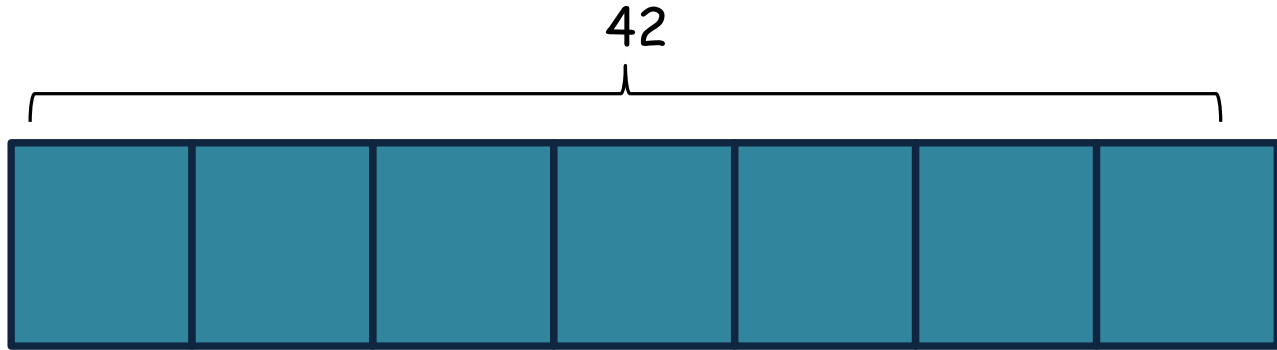
42

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Multiplying Fractions by Whole Numbers Using Tape Diagrams

Step 4:   Determine how much 1 unit is.

42

?
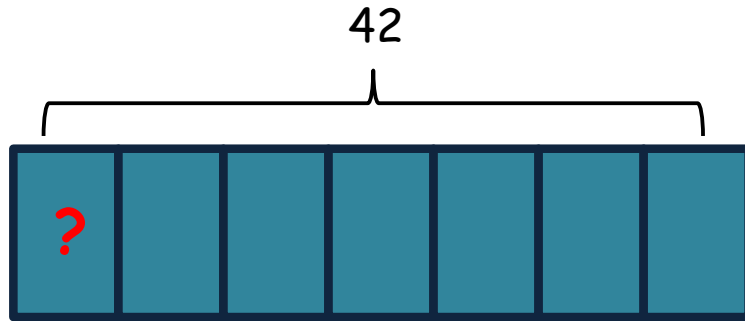
7 units = 42

1 unit = 42 ÷ 7

1 unit = 6

$\frac{1}{7}$ of 42 = 6

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Multiplying Fractions by Whole Numbers Using Tape Diagrams

Step 5: Now that you know how much 1 unit is, you can now solve the problem.

What is $\frac{3}{7}$ of 42?

We know that 1 seventh of 42 is equal to 6, how about 3 sevenths of 42?

Multiplication of Fractions and Whole Numbers

Multiplying Fractions by Whole Numbers Using Tape Diagrams

42

?	?	?				
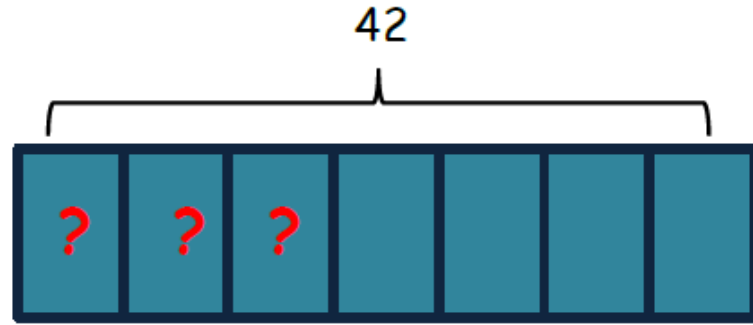
If 1 unit = 6, then,
3 units = 3 × 6
3 units = 18

Therefore, $\frac{3}{7}$ of 42 = 18.

Multiplying Fractions by Whole Numbers Using Tape Diagrams

b. Using a tape diagram, find $\frac{5}{6}$ of 54.

54

| ? | ? | ? | ? | ? | |
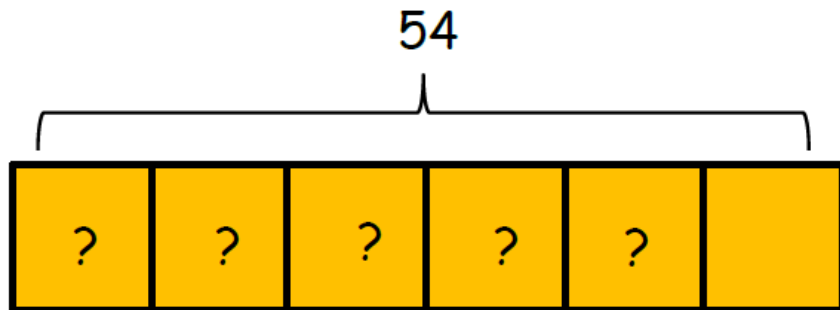
6 units = 54
1 unit = 54 ÷ 6
1 unit = 9
$\frac{1}{6}$ of 54 = 9

If 1 unit = 9, then:
5 units = 5 × 9
5 units = 45
$\frac{5}{6}$ of 54 = 45

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Sample Problem 2: Solve the following using a tape diagram.

1. $\frac{3}{7}$ of 49 = ?

2. $\frac{3}{8} \times 64$ = ?

Multiplication of Fractions and Whole Numbers

Sample Problem 2: Solve the following using a tape diagram.

Solution:

1. $\frac{3}{7}$ of 49 = 21

49

7 units = 49
1 unit = 49 ÷ 7
1 unit = 7
$\frac{1}{7}$ of 49 = 7

If 1 unit = 7, then:
3 units = 3 × 7
3 units = 21
$\frac{3}{7}$ of 49 = 21

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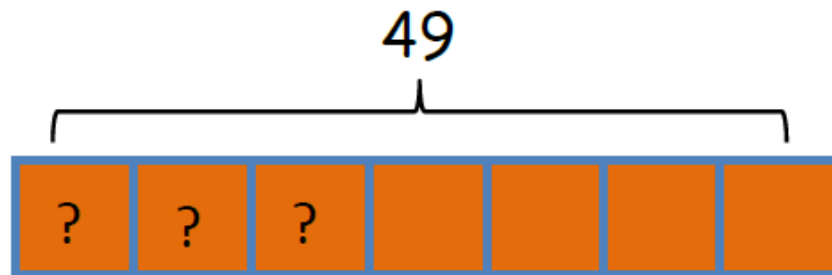
Multiplication of Fractions and Whole Numbers

Sample Problem 2: Solve the following using a tape diagram.

Solution:

2. $\frac{3}{8} \times 64 = 24$

64
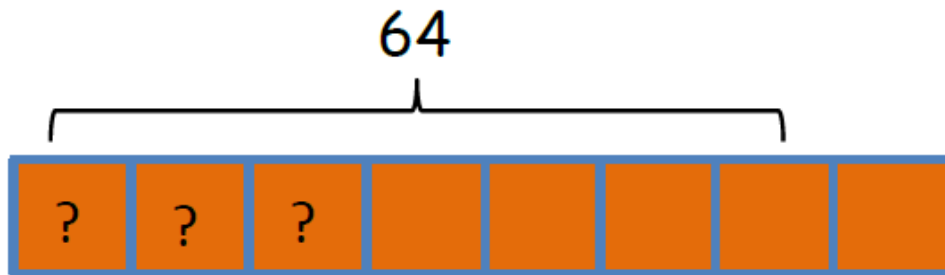
8 units = 64

1 unit = 64 ÷ 8

1 unit = 8

$\frac{1}{8}$ of 64 = 8

If 1 unit = 8, then:

3 units = 3 × 8

3 units = 24

$\frac{3}{8}$ of 64 = 24

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Fraction Multiplication as Repeated Addition

It is a known fact that multiplication is repeated addition. For instance:

$$7 \times 6 = 42$$

is the same as

$$7 + 7 + 7 + 7 + 7 + 7 = 42$$

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Fraction Multiplication as Repeated Addition

How does repeated addition work for multiplication of fractions and whole number? For instance:

$$\frac{2}{3} \times 9$$
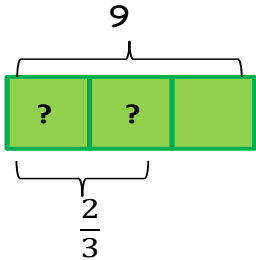
can be expressed as:

9 duplicates of 2 thirds

or

2 thirds added altogether 9 times

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Multiplication of Fractions and Whole Numbers

Using a Tape Diagram	Repeated Addition
$\frac{2}{3} \times 9 = ?$	$\frac{2}{3} \times 9 = ?$

Using a Tape Diagram

$$\frac{2}{3} \times 9 = ?$$

9

?	?	

$\frac{2}{3}$

3 units = 9

1 unit = 9 ÷ 3

1 unit = $\frac{9}{3}$

1 unit = 3

2 units = 2 × $\frac{9}{3}$

2 units = $\frac{9}{3} + \frac{9}{3}$

2 units = $\frac{18}{3}$

2 units = 6

Repeated Addition

$$\frac{2}{3} \times 9 = ?$$

$$= \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$$

$$= \frac{2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2}{3}$$

$$\frac{2}{3} \times 9 = \frac{18}{3}$$

$$\frac{2}{3} \times 9 = 6$$

Multiplication of Fractions and Whole Numbers

In general, multiplying fractions and whole numbers is just a piece of cake! The visual models such as arrays and tape diagrams, plus the repeated addition discussed in this lesson gives you a better understanding of how and what happens when fractions and whole numbers are multiplied.

So how is it done?

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Multiplication of Fractions and Whole Numbers

Example: What is $\frac{3}{4}$ of 44?

Step 1:   Remember that whole numbers when expressed as fractions has a denominator of 1.

$$44 = \frac{44}{1}$$

Therefore, the multiplication sentence is: $\frac{3}{4} \times \frac{44}{1}$

Multiplication of Fractions and Whole Numbers

Step 2: Multiply the numerators of the fractions.
Multiply the denominators of the fractions.

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$ $$\longrightarrow$$ $$\frac{3}{4} \times \frac{44}{1} = \frac{(3)(44)}{(4)(1)}$$

At this point, you have 2 options; you can either multiply the numerators and the denominators of the fraction before simplifying the answer, or you can also reduce the fraction first before multiplying. Either way will give the same answer.

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Multiplication of Fractions and Whole Numbers

Multiplying the numerators and denominators of the fraction before simplifying:

Step 3:

$$\frac{3}{4} \times \frac{44}{1} = \frac{(3)(44)}{(4)(1)}$$

$$= \frac{132}{4}$$

$$\frac{3}{4} \times \frac{44}{1} = 33$$

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Multiplication of Fractions and Whole Numbers

Reducing the fractions by removing the common factors, before simplifying.

Step 3:

$$\frac{3}{4} \times \frac{44}{1} = \frac{(3)(44)}{(4)(1)}$$

$$= \frac{(3)(4)(11)}{(4)(1)}$$

$$\frac{3}{4} \times \frac{44}{1} = \frac{(3)(11)}{1}$$

$$= \frac{33}{1}$$

$$= 33$$

Therefore, $\frac{3}{4}$ of 44 = 33.

Sample Problem 3: Multiply the following using two ways. The first one is done for you.

1. $\frac{4}{5} \times 35 =$

$\frac{4}{5} \times \frac{35}{1} = \frac{(4)(35)}{(5)(1)}$ $= \frac{140}{5}$ $= 28$	$\frac{4}{5} \times \frac{35}{1} = \frac{(4)(35)}{(5)(1)}$ $= \frac{(4)(5)(7)}{(5)(1)}$ $= \frac{(4)(7)}{1}$ $= \frac{28}{1}$ $= 28$

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Multiplication of Fractions and Whole Numbers

2. $\dfrac{2}{7} \times 42 =$

| | |
| | |

3. $\dfrac{3}{8} \times 64 =$

| | |
| | |

Multiplication of Fractions and Whole Numbers

Solution:

2. $\dfrac{2}{7} \times 42 =$

$$\dfrac{2}{7} \times \dfrac{42}{1} = \dfrac{(2)(42)}{(7)(1)}$$
$$= \dfrac{84}{7}$$
$$= 12$$

$$\dfrac{2}{7} \times \dfrac{42}{1} = \dfrac{(2)(42)}{(7)(1)}$$
$$= \dfrac{(2)(7)(6)}{(7)(1)}$$
$$= \dfrac{(2)(6)}{1}$$
$$= \dfrac{12}{1}$$
$$= 12$$

Multiplication of Fractions and Whole Numbers

3. $\dfrac{3}{8} \times 64 =$

$\dfrac{3}{8} \times \dfrac{64}{1} = \dfrac{(3)(64)}{(8)(1)}$ $= \dfrac{192}{8}$ $= 24$	$\dfrac{3}{8} \times \dfrac{64}{1} = \dfrac{(3)(64)}{(8)(1)}$ $= \dfrac{(3)(8)(8)}{(8)(1)}$ $= \dfrac{(3)(8)}{1}$ $= \dfrac{24}{1}$ $= 24$

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Multiplication of Fractions and Whole Numbers - Word Problems

This time you will be dealing with problems involving multiplication of fractions and whole numbers in real life situations. Arrays and tape diagrams can also be used to solve these problems.

Example:

Nica is 32 years old. She spent $\frac{3}{8}$ of her life in Chicago. For how many years did Nica live in Chicago?
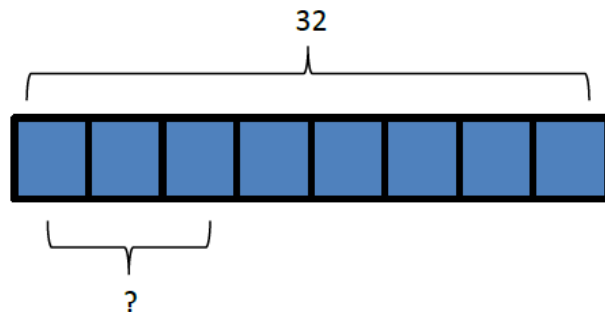
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Example:

Nica is 32 years old. She spent $\frac{3}{8}$ of her life in Chicago. For how many years did Nica live in Chicago?

a. Use a tape diagram to show your solution:

$\frac{3}{8}$ of 32 = ?

32

?

8 units = 32
1 unit = 32 ÷ 8
1 unit = 4
$\frac{1}{8}$ of 32 = 4

If 1 unit = 4, then:
3 units = 4 x 3
3 units = 12
$\frac{3}{8}$ of 32 = 12

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Example:

Nica is 32 years old. She spent $\frac{3}{8}$ of her life in Chicago. For how many years did Nica live in Chicago?

b. Multiply directly to show the solution.

$$\frac{3}{8} \times \frac{32}{1} = \frac{(3)(32)}{(8)(1)}$$

$$= \frac{96}{8}$$

$$= 12$$

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Sample Problem 4:

Patty baked 4 dozens of cookies. $\frac{5}{6}$ are choco-chip cookies, and the rest are butter cookies. How many choco-chip and butter cookies are there? Show your solution using a tape diagram and using multiplication.
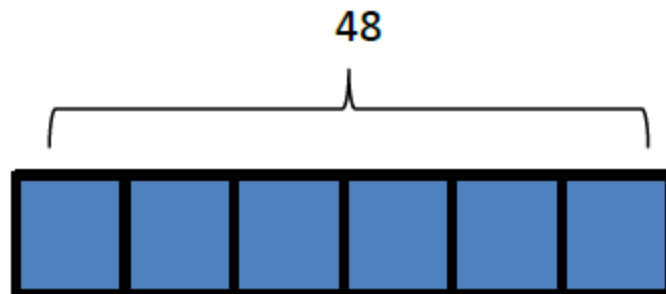
Multiplication of Fractions and Whole Numbers

Solution:

a. 1 dozen = 12

4 dozens = 48

$\frac{5}{6}$ of 48 = ?

48

6 units = 48
1 unit = 48 ÷ 6
1 unit = 8
$\frac{1}{6}$ of 48 = 8

If 1 unit = 8, then:
5 units = 5 x 8
5 units = 40
$\frac{5}{6}$ of 48 = 40

There are 40 choco-chip cookies and 8 butter cookies.

Multiplication of Fractions and Whole Numbers

b. Multiply directly to show the solution.

$$\frac{5}{6} \times \frac{48}{1} = \frac{(5)(48)}{(6)(1)}$$
$$= \frac{240}{6}$$
$$= 40$$

There are 40 choco-chip cookies and 8 butter cookies.

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