

Multiplication of Fractions and Whole Number

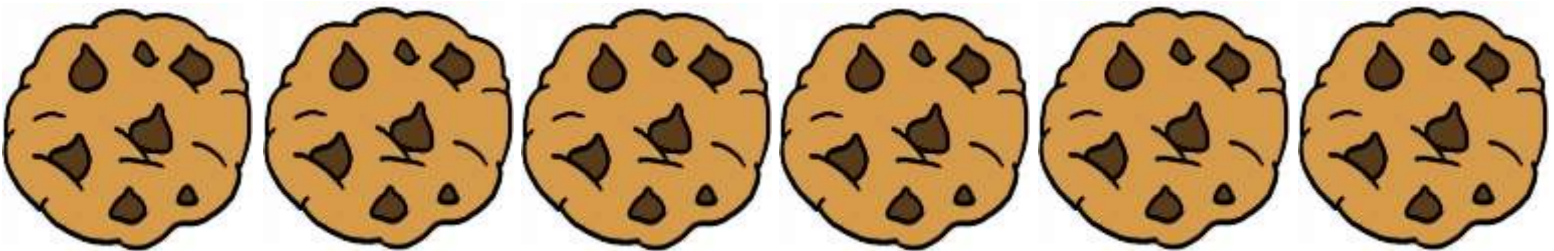
Finding Fractions of a Set

A **fraction** is a division of the **numerator** by the **denominator**. A fraction can be expressed as division and vice versa.

FRACTION	DIVISION
$\frac{a}{b}$	$a \div b$

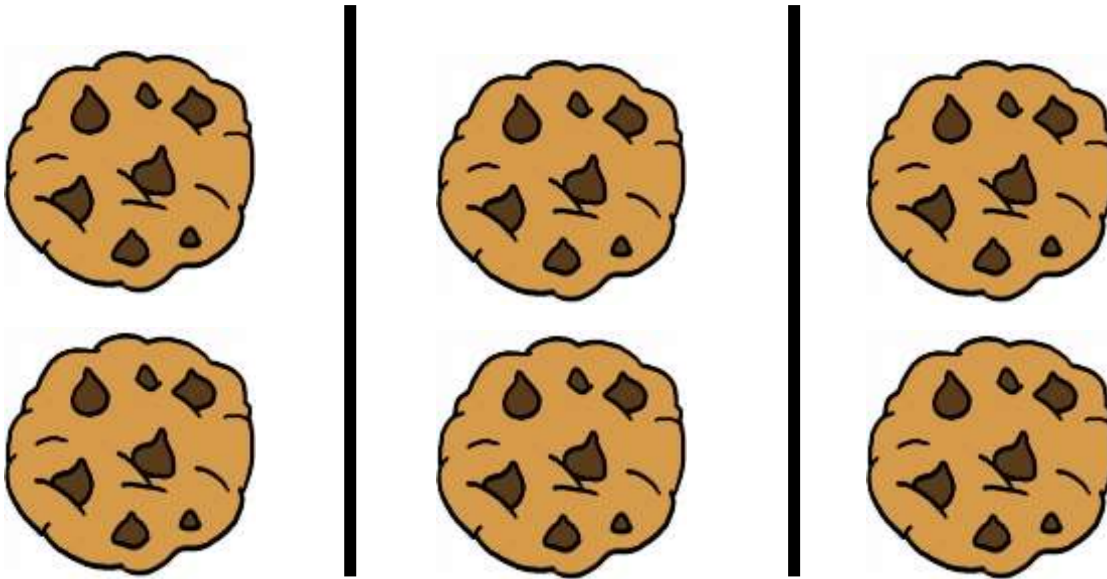
In this lesson, you will multiply fractions and whole numbers using arrays of figures or shapes to find fractions of a set.

If there are 6 cookies and you are to divide it into 3 parts equally, how many cookies will each part have?



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Dividing the cookies, you'll get:



The division sentence for the given problem is:

$$6 \div 3 = 2$$

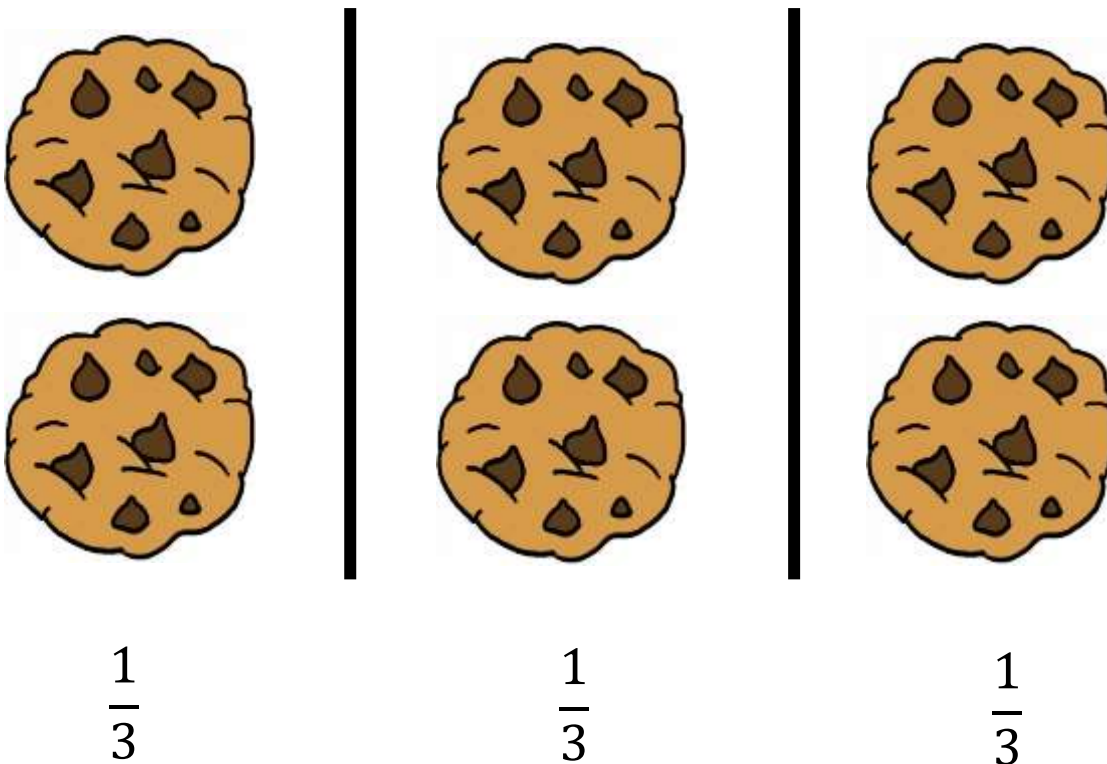
The division sentence expressed as a fraction is:

$$\frac{6}{3} = 2$$

If you want to know 1 third ($\frac{1}{3}$) of the cookies, how many cookies will there be in each part?

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Each part/group of cookies divided equally is one-third of the entire cookies.



This shows that when you divide 6 by 3, there are 2 in each group, therefore:

$\frac{1}{3}$ of 6 cookies is equal to 2 cookies.

$\frac{2}{3}$ of 6 cookies is equal to 4 cookies.

$\frac{3}{3}$ of 6 cookies is equal to 6 cookies.

Multiplication of Fractions and Whole Number

Example 1: What is $\frac{1}{3}$ of 12?

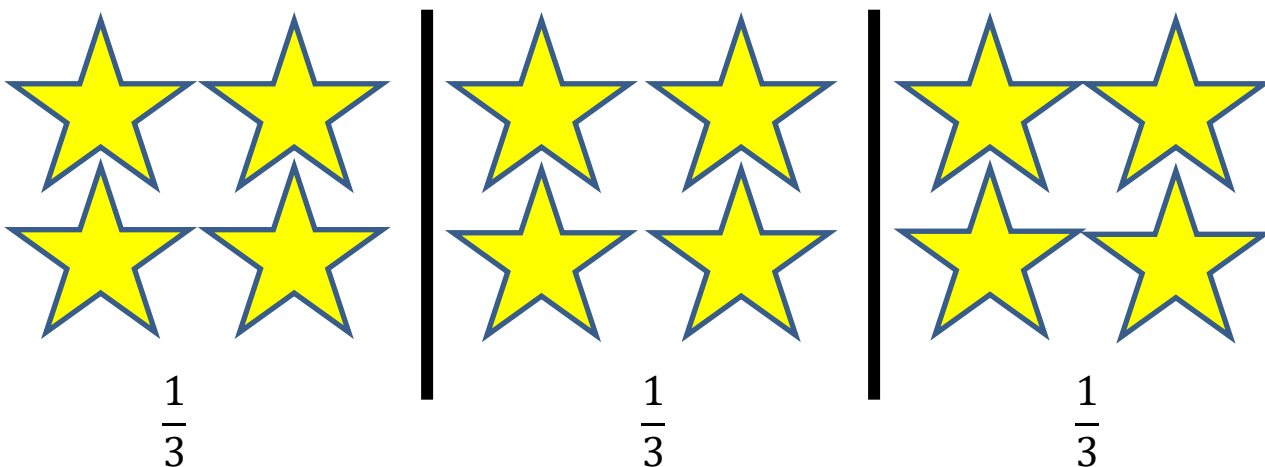
Make an array of 12 shapes (of the same kind) and divide it equally into three parts.



The division sentence expressed as a fraction is:

$$\frac{12}{3} = 4$$

Each part/group of the stars divided equally is 1 third of the 24 stars.



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Therefore,

$$\frac{1}{3} \text{ of } 12 = 4$$

$$\frac{2}{3} \text{ of } 12 = 8$$

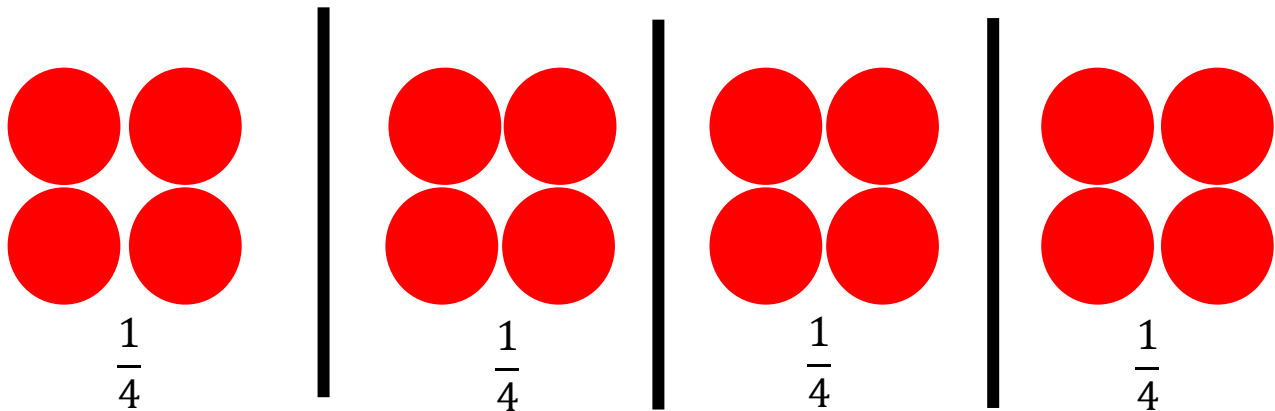
$$\frac{3}{3} \text{ of } 12 = 12$$

12 divided into 3 equal parts,
each part or group is equal to 3.

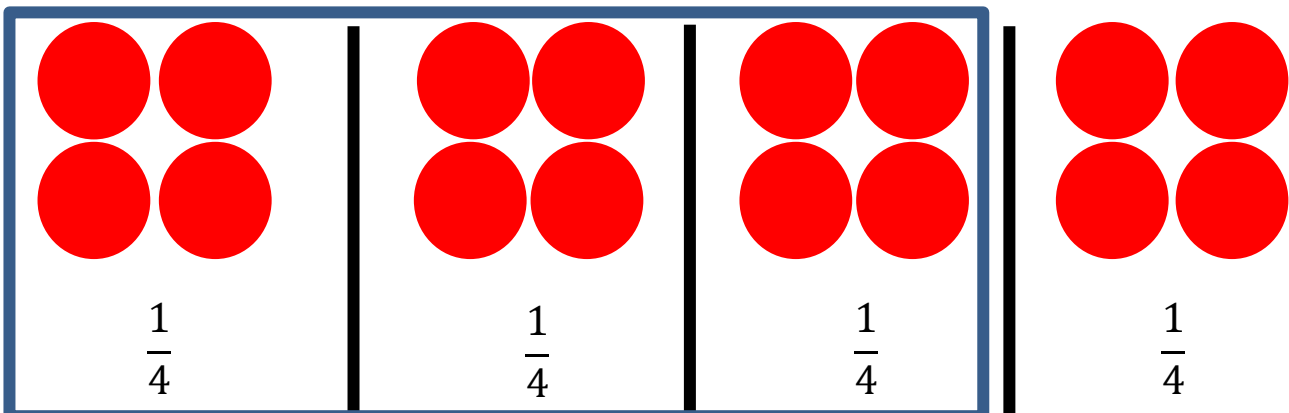
$$\frac{1}{3} \text{ of } 12 = 4$$

Example 2: What is $\frac{3}{4}$ of 16?

Draw 16 any shape of the same kind, and divide it equally into 6 groups.



Here is three-fourths of 16:

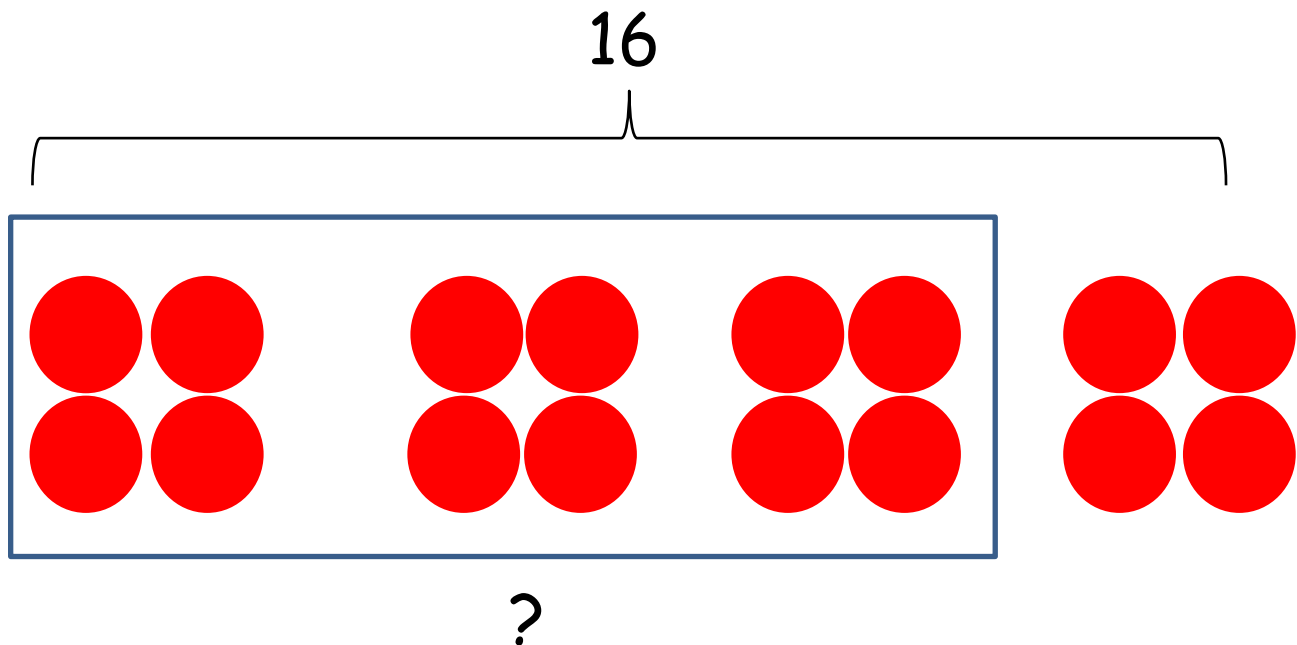


Therefore, $\frac{3}{4}$ of 16 = 12.

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The problem can also be solved this way.

What is $\frac{3}{4}$ of 16?



$$16 \div 4 = 4$$
$$\frac{1}{4} \text{ of } 16 = 4$$

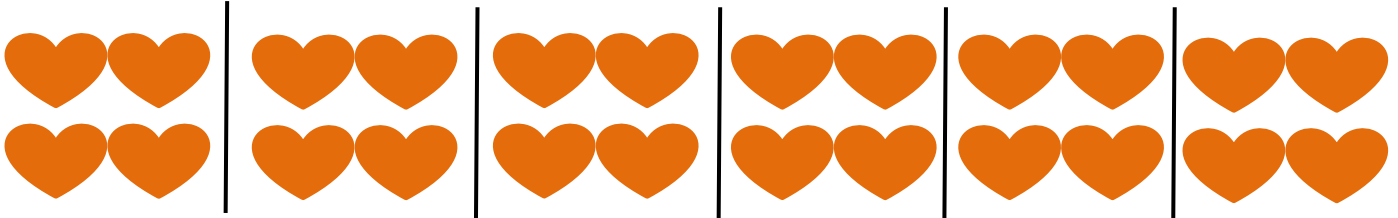
If 16 is divided equally into 4 parts, each part is equal to 4. Since you are looking for 3 parts and you know that each part is 4, then:

$$3 \times 4 = 12$$

$$\frac{3}{4} \text{ of } 16 = 12$$

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Example 3: Find the value of the following.



a. $\frac{1}{6}$ of 24 = ?

Divide 24 into 6 equal parts: $24 \div 6 = 4$

Each part is 1 sixth of 24.

Therefore, $\frac{1}{6}$ of 24 = 4

b. $\frac{3}{6}$ of 24 = ?

Each part is 1 sixth of 24.

$\frac{1}{6}$ of 24 = 4

Therefore, $\frac{3}{6}$ of 24 = 12.

c. $\frac{5}{6}$ of 24 = ?

Each part is 1 sixth of 24.

$\frac{1}{6}$ of 24 = 4

Therefore, $\frac{5}{6}$ of 24 = 20.

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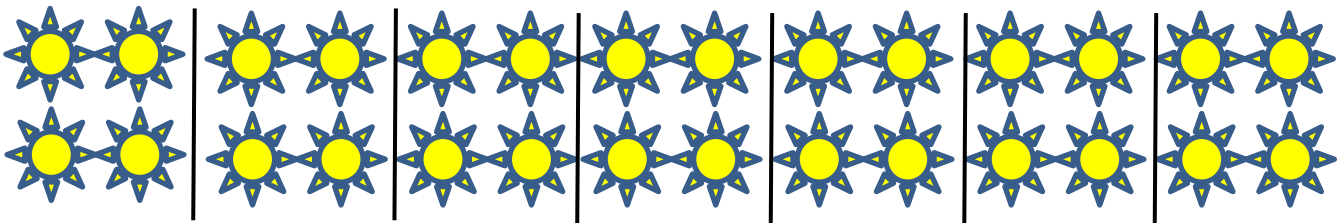
Sample Problem 1:

A. Draw an array of circles to find the value of the following.

1. $\frac{1}{5}$ of 35 = ?

2. $\frac{4}{6}$ of 36 = ?

B. Find the value of the following.



1. $\frac{1}{7}$ of 28 = ?

2. $\frac{4}{7}$ of 28 = ?

3. $\frac{3}{7}$ of 28 = ?

4. $\frac{6}{7}$ of 28 = ?

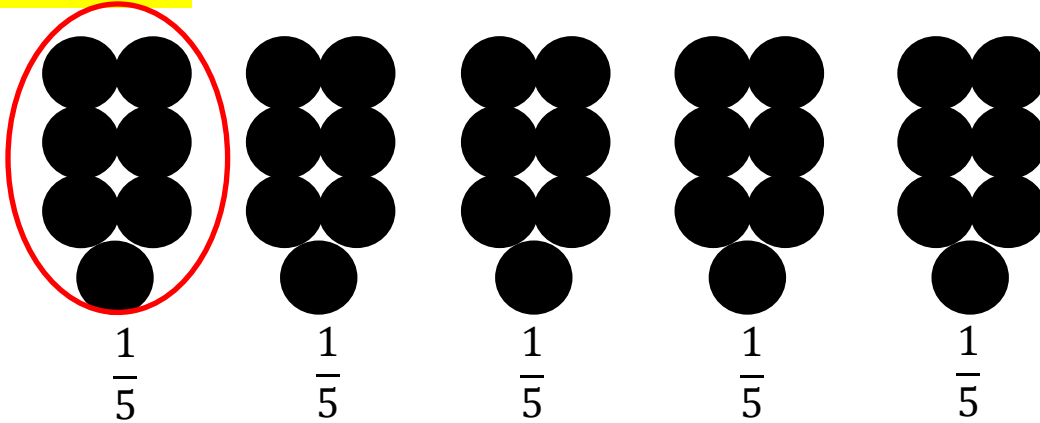
5. $\frac{5}{7}$ of 28 = ?

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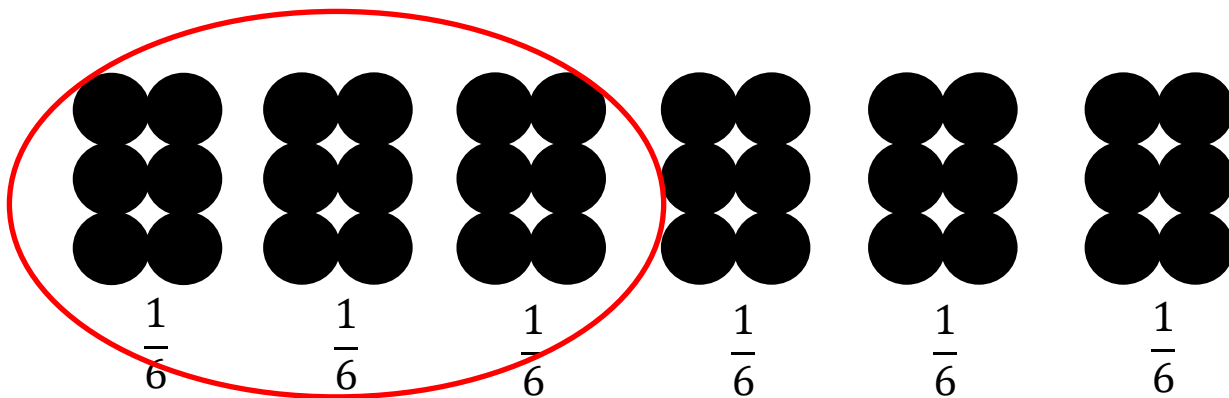
Solution:

A. Draw an array of circles to find the value of the following.

1. $\frac{1}{5}$ of 35 = 7

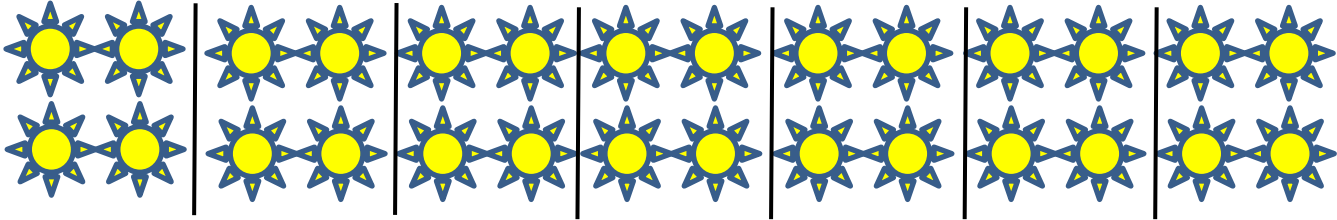


2. $\frac{4}{6}$ of 36 = 18



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B. Find the value of the following.



1. $\frac{1}{7}$ of 28 = 4

2. $\frac{4}{7}$ of 28 = 16

3. $\frac{3}{7}$ of 28 = 12

4. $\frac{6}{7}$ of 28 = 24

5. $\frac{5}{7}$ of 28 = 20

Multiplication of Fractions and Whole Number

Multiplying Fractions by Whole Numbers Using Tape Diagrams

Previously, you created arrays of shapes to show how to find a part of a whole, which is exactly the same as multiplying a fraction to a whole number, example:

$$\frac{1}{4} \text{ of } 8$$

is the same as

$$\frac{1}{4} \times 8$$

This time you will multiply fractions and whole numbers using tape diagrams. How is it done?

Examples:

a. Using a tape diagram, find $\frac{3}{7}$ of 42.

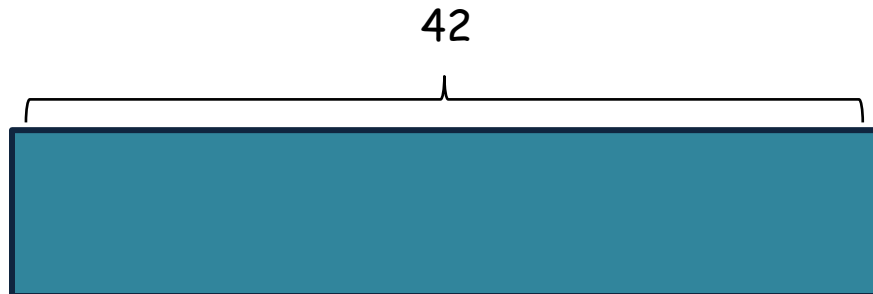
What is $\frac{3}{7}$ of 42?

Step 1: Here, you need to find 3 sevenths of 42. First, determine the whole in the given problem.

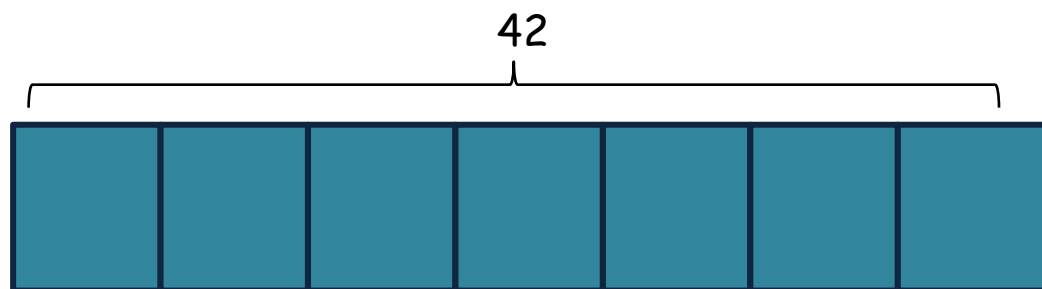
$$\frac{3}{7} \text{ of } 42 = ? \quad 42 \text{ is the whole!}$$

Multiplication of Fractions and Whole Number

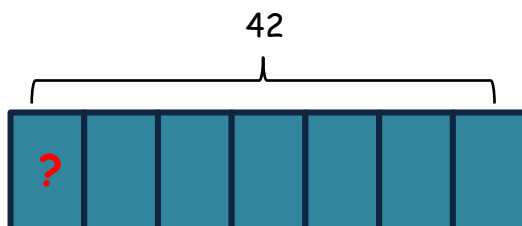
Step 2: Draw a bar that will represent the whole.



Step 3: Since we want $\frac{3}{7}$ of 42, the denominator in $\frac{3}{7}$ tells us how many units we need to cut the tape diagram. Thus, we need to cut the bar into 7 equal parts.



Step 4: Determine how much 1 unit is.



$$\begin{aligned} 7 \text{ units} &= 42 \\ 1 \text{ unit} &= 42 \div 7 \\ 1 \text{ unit} &= 6 \\ \frac{1}{7} \text{ of } 42 &= 6 \end{aligned}$$

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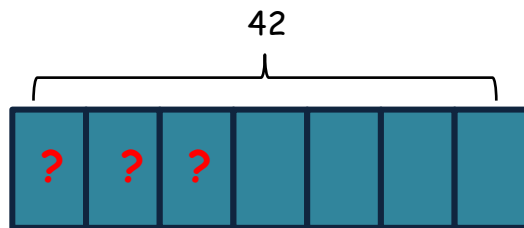
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Step 5: Now that you know how much 1 unit is, you can now solve the problem.

What is $\frac{3}{7}$ of 42?

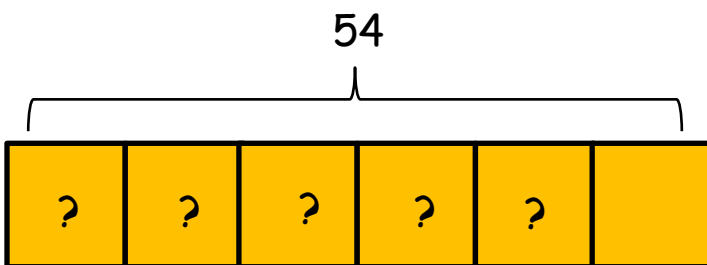
We know that 1 seventh of 42 is equal to 6, how about 3 sevenths of 42?



If 1 unit = 6, then,
3 units = 3×6
3 units = 18

Therefore, $\frac{3}{7}$ of 42 = 18.

b. Using a tape diagram, find $\frac{5}{6}$ of 54.



6 units = 54
1 unit = $54 \div 6$
1 unit = 9
 $\frac{1}{6}$ of 54 = 9

If 1 unit = 9, then:
5 units = 5×9
5 units = 45
 $\frac{5}{6}$ of 54 = 45

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Sample Problem 2: Solve the following using a tape diagram.

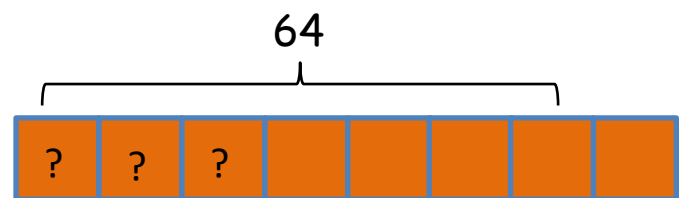
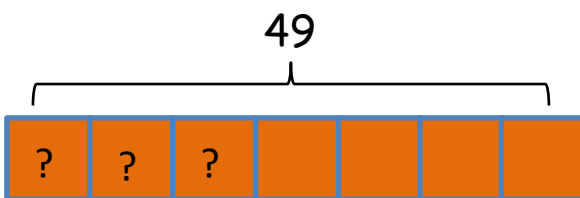
1. $\frac{3}{7}$ of 49 = ?

2. $\frac{3}{8} \times 64 = ?$

Solution:

1. $\frac{3}{7}$ of 49 = 21

2. $\frac{3}{8} \times 64 = 24$



7 units = 49
 1 unit = $49 \div 7$
 1 unit = 7
 $\frac{1}{7}$ of 49 = 7

If 1 unit = 7, then:
 3 units = 3×7
 3 units = 21
 $\frac{3}{7}$ of 49 = 21

8 units = 64
 1 unit = $64 \div 8$
 1 unit = 8
 $\frac{1}{8}$ of 64 = 8

If 1 unit = 8, then:
 3 units = 3×8
 3 units = 24
 $\frac{3}{8}$ of 64 = 24

Multiplication of Fractions and Whole Number

Fraction Multiplication as Repeated Addition

It is a known fact that multiplication is repeated addition. For instance:

$$7 \times 6 = 42$$

is the same as

$$7 + 7 + 7 + 7 + 7 + 7 = 42$$

How does repeated addition work for multiplication of fractions and whole number? For instance:

$$\frac{2}{3} \times 9$$

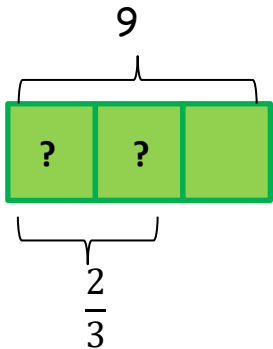
can be expressed as:

9 duplicates of
2 thirds

or

2 thirds added altogether 9
times

Multiplication of Fractions and Whole Number

Using a Tape Diagram	Repeated Addition
<p style="text-align: center;">$\frac{2}{3} \times 9 = ?$</p> <div style="text-align: center;">  </div> <p>3 units = 9 1 unit = $9 \div 3$ 1 unit = $\frac{9}{3}$ 1 unit = 3 2 units = $2 \times \frac{9}{3}$ 2 units = $\frac{9}{3} + \frac{9}{3}$ 2 units = $\frac{18}{3}$ 2 units = 6</p>	<p style="text-align: center;">$\frac{2}{3} \times 9 = ?$</p> $= \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$ $= \frac{2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2}{3}$ <div style="text-align: center;"> <p>$\frac{2}{3} \times 9 = \frac{18}{3}$</p> <p>$\frac{2}{3} \times 9 = 6$</p> </div>

Multiplication of Fractions and Whole Number

Multiplication of Fractions and Whole Numbers

In general, multiplying fractions and whole numbers is just a piece of cake! The visual models such as arrays and tape diagrams, plus the repeated addition discussed in this lesson gives you a better understanding of how and what happens when fractions and whole numbers are multiplied.

So how is it done?

Example: What is $\frac{3}{4}$ of 44?

Step 1: Remember that whole numbers when expressed as fractions has a denominator of 1.

$$44 = \frac{44}{1}$$

Therefore, the multiplication sentence is: $\frac{3}{4} \times \frac{44}{1}$

Step 2: Multiply the numerators of the fractions.
Multiply the denominators of the fractions.

$$\boxed{\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}} \quad \longrightarrow \quad \boxed{\frac{3}{4} \times \frac{44}{1} = \frac{(3)(44)}{(4)(1)}}$$

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At this point, you have 2 options; you can either multiply the numerators and the denominators of the fraction before simplifying the answer, or you can also reduce the fraction first before multiplying. Either way will give the same answer.

Multiplying the numerators and denominators of the fraction before simplifying:

Step 3:

$$\begin{aligned}\frac{3}{4} \times \frac{44}{1} &= \frac{(3)(44)}{(4)(1)} \\ &= \frac{132}{4} \\ \frac{3}{4} \times \frac{44}{1} &= 33\end{aligned}$$

Reducing the fractions by removing the common factors, before simplifying.

Step 3:

$$\begin{aligned}\frac{3}{4} \times \frac{44}{1} &= \frac{(3)(44)}{(4)(1)} \\ &= \frac{(3)(\cancel{4})(11)}{(\cancel{4})(1)}\end{aligned}$$

$$\begin{aligned}\frac{3}{4} \times \frac{44}{1} &= \frac{(3)(11)}{1} \\ &= \frac{33}{1} \\ &= 33\end{aligned}$$

Therefore, $\frac{3}{4}$ of 44 = 33.

Multiplication of Fractions and Whole Number

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Sample Problem 3: Multiply the following using two ways. The first one is done for you.

1. $\frac{4}{5} \times 35 =$

$$\begin{aligned}\frac{4}{5} \times \frac{35}{1} &= \frac{(4)(35)}{(5)(1)} \\ &= \frac{140}{5} \\ &= 28\end{aligned}$$

$$\begin{aligned}\frac{4}{5} \times \frac{35}{1} &= \frac{(4)(35)}{(5)(1)} \\ &= \frac{(4)(5)(7)}{(5)(1)} \\ &= \frac{(4)(7)}{1} \\ &= 28\end{aligned}$$

2. $\frac{2}{7} \times 42 =$

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3. $\frac{3}{8} \times 64 =$

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Multiplication of Fractions and Whole Number

Solution:

2. $\frac{2}{7} \times 42 =$

$$\begin{aligned}\frac{2}{7} \times \frac{42}{1} &= \frac{(2)(42)}{(7)(1)} \\ &= \frac{84}{7} \\ &= \mathbf{12}\end{aligned}$$

$$\begin{aligned}\frac{2}{7} \times \frac{42}{1} &= \frac{(2)(42)}{(7)(1)} \\ &= \frac{(2)(7)(6)}{(7)(1)} \\ &= \frac{(2)(6)}{1} \\ &= \frac{12}{1} \\ &= \mathbf{12}\end{aligned}$$

3. $\frac{3}{8} \times 64 =$

$$\begin{aligned}\frac{3}{8} \times \frac{64}{1} &= \frac{(3)(64)}{(8)(1)} \\ &= \frac{192}{8} \\ &= \mathbf{24}\end{aligned}$$

$$\begin{aligned}\frac{3}{8} \times \frac{64}{1} &= \frac{(3)(64)}{(8)(1)} \\ &= \frac{(3)(8)(8)}{(8)(1)} \\ &= \frac{(3)(8)}{1} \\ &= \frac{24}{1} \\ &= \mathbf{24}\end{aligned}$$

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Multiplication of Fractions and Whole Numbers - Word Problems

This time you will be dealing with problems involving multiplication of fractions and whole numbers in real life situations. Arrays and tape diagrams can also be used to solve these problems.

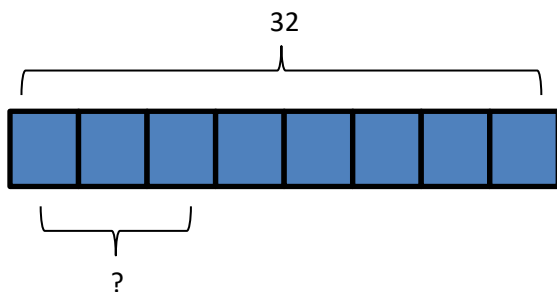
Example:

Nica is 32 years old. She spent $\frac{3}{8}$ of her life in Chicago. For how many years did Nica live in Chicago?

a. Use a tape diagram to show your solution:

Solution:

$$\frac{3}{8} \text{ of } 32 = ?$$



$$\begin{aligned} 8 \text{ units} &= 32 \\ 1 \text{ unit} &= 32 \div 8 \\ 1 \text{ unit} &= 4 \\ \frac{1}{8} \text{ of } 32 &= 4 \end{aligned}$$

$$\begin{aligned} \text{If } 1 \text{ unit} &= 4, \text{ then:} \\ 3 \text{ units} &= 4 \times 3 \\ 3 \text{ units} &= 12 \\ \frac{3}{8} \text{ of } 32 &= 12 \end{aligned}$$

b. Multiply directly to show the solution.

$$\begin{aligned} \frac{3}{8} \times \frac{32}{1} &= \frac{(3)(32)}{(8)(1)} \\ &= \frac{96}{8} \\ &= 12 \end{aligned}$$

Multiplication of Fractions and Whole Number

Sample Problem 4:

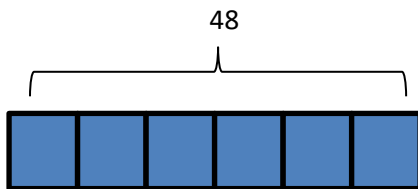
Patty baked 4 dozens of cookies. $\frac{5}{6}$ are choco-chip cookies, and the rest are butter cookies. How many choco-chip and butter cookies are there? Show your solution using a tape diagram and using multiplication.

Solution:

a. 1 dozen = 12

4 dozens = 48

$\frac{5}{6}$ of 48 = ?



6 units = 48
 1 unit = $48 \div 6$
 1 unit = 8
 $\frac{1}{6}$ of 48 = 8

If 1 unit = 8, then:
 5 units = 5×8
 5 units = 40
 $\frac{5}{6}$ of 48 = 40

There are 40 choco-chip cookies and 8 butter cookies.

b. Multiply directly to show the solution.

$$\frac{5}{6} \times \frac{48}{1} = \frac{(5)(48)}{(6)(1)}$$

$$= \frac{240}{6}$$

= 40 There are 40 choco-chip cookies and 8 butter cookies.